SOLUTIONS : DIFFERENTIATING VECTOR-VALUED FUNCTIONS

SECTION ONE

1(a) $\underset{\sim}{r}'(t) = e^t \underset{\sim}{i} - 2e^{-2t} \underset{\sim}{j}$

At the point $(1,1)$, $t=0$ and $\underset{\sim}{r}'(0) = \langle 1, -2 \rangle$

so the tangent line is $\underline{\underline{\langle 1, 1 \rangle + s \langle 1, -2 \rangle}}$ $s \in \mathbb{R}$

(b) At $(0,2,1)$, $t=1$. $\underset{\sim}{r}'(t) = \langle \frac{1}{t}, \frac{1}{\sqrt{t}}, 2t \rangle$

so $\underset{\sim}{r}'(1) = \langle 1, 1, 2 \rangle$

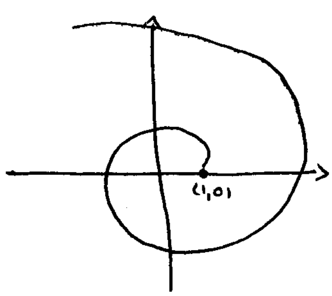
and the tangent line is $\underline{\underline{\langle 0, 2, 1 \rangle + s \langle 1, 1, 2 \rangle}}$ $s \in \mathbb{R}$

2 L is $\langle \cos t, \sin t, t \rangle + s \langle -\sin t, \cos t, 1 \rangle$ $s \in \mathbb{R}$

$= \langle \cos t - s \sin t, \sin t + s \cos t, t + s \rangle$

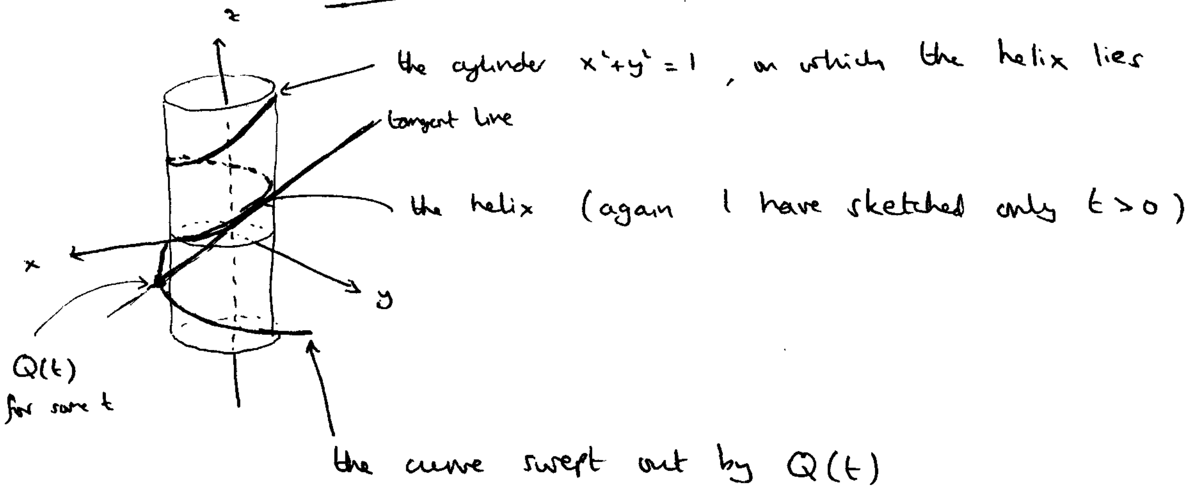
This meets the xy-plane when $s = -t$, so

$$Q(t) = \langle \cos t + t \sin t, \sin t - t \cos t, 0 \rangle.$$



This looks like this (for $t > 0$) — I used Mathematica here.

I have sketched only $t > 0$, for clarity



SECTION TWO :

On a sphere, $\|\mathbf{r}(t)\|^2 = \mathbf{r}(t)\cdot\mathbf{r}(t)$ is constant.

Thus $\frac{d}{dt}\left(\mathbf{r}(t)\cdot\mathbf{r}(t)\right) = 0$, so $2\,\mathbf{r}(t)\cdot\mathbf{r}'(t) = 0$

$\Rightarrow$ $\mathbf{r}(t)$ is perpendicular to $\mathbf{r}'(t)$.

Conversely, if $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are always perpendicular then $\frac{d}{dt}\left(\|\mathbf{r}(t)\|^2\right) = 0$, and so the particle moves on a sphere of constant radius about the origin.

SECTION THREE

(1) $\mathbf{F} = m\mathbf{a}$

(2) $\mathbf{F}$ is parallel to $\mathbf{r}$, so $\mathbf{a}$ is parallel to $\mathbf{r}$.

But $\frac{d}{dt}(\mathbf{h}) = \frac{d}{dt}(\mathbf{r}\times\mathbf{r}')$

$$= \underbrace{\cancel{\mathbf{r}'\times\mathbf{r}'}}_{\text{dies as } \mathbf{r}' \text{ is parallel to } \mathbf{r}'} + \underbrace{\cancel{\mathbf{r}\times\mathbf{r}''}}_{\text{dies as } \mathbf{r}''=\mathbf{a} \text{ is parallel to } \mathbf{r}} = \mathbf{0}$$

(3) $\mathbf{r}(t)$ is always perpendicular to $\mathbf{h}$ (which is constant) so the particle moves in the plane through the origin perp. to $\mathbf{h}$.