12 Review

• CONCEPT CHECK •

1. (a) A double Riemann sum of f is $\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$, where ΔA is the area of each subrectangle and (x_{ij}^*, y_{ij}^*) is a sample point in each subrectangle. If $f(x,y) \geq 0$, this sum represents an approximation to the volume of the solid that lies above the rectangle R and below the graph of f.

   (b) $\iint_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\,\Delta A$

   (c) If $f(x,y) \geq 0$, $\iint_R f(x,y)\,dA$ represents the volume of the solid that lies above the rectangle R and below the surface $z = f(x,y)$. If f takes on both positive and negative values, $\iint_R f(x,y)\,dA$ is the difference of the volume above R but below the surface $z = f(x,y)$ and the volume below R but above the surface $z = f(x,y)$.

   (d) We usually evaluate $\iint_R f(x,y)\,dA$ as an iterated integral according to Fubini's Theorem (see Theorem 12.2.4).

   (e) The Midpoint Rule for Double Integrals says that we approximate the double integral $\iint_R f(x,y)\,dA$ by the double Riemann sum $\sum_{i=1}^{m}\sum_{j=1}^{n} f(\overline{x}_i, \overline{y}_j)\,\Delta A$ where the sample points $(\overline{x}_i, \overline{y}_j)$ are the centers of the subrectangles.

   (f) $f_{\text{ave}} = \dfrac{1}{A(R)} \displaystyle\iint_R f(x,y)\,dA$ where $A(R)$ is the area of R.

2. (a) See (1) and (2) and the accompanying discussion in Section 12.3.

   (b) See (3) and the preceding discussion in Section 12.3.

   (c) See (5) and the preceding discussion in Section 12.3.

   (d) See (6)–(11) in Section 12.3.

3. We may want to change from rectangular to polar coordinates in a double integral if the region R of integration is more easily described in polar coordinates. To accomplish this, we use $\iint_R f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$ where R is given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$.

4. (a) $m = \iint_D \rho(x,y)\,dA$

(b) $M_x = \iint_D y\rho(x,y)\,dA$, $M_y = \iint_D x\rho(x,y)\,dA$

(c) The center of mass is $(\overline{x},\overline{y})$ where $\overline{x} = \dfrac{M_y}{m}$ and $\overline{y} = \dfrac{M_x}{m}$.

(d) $I_x = \iint_D y^2\rho(x,y)\,dA$, $I_y = \iint_D x^2\rho(x,y)\,dA$, $I_0 = \iint_D (x^2+y^2)\rho(x,y)\,dA$

5. (a) $P(a \le X \le b, c \le Y \le d) = \int_a^b \int_c^d f(x,y)\,dy\,dx$

(b) $f(x,y) \ge 0$ and $\iint_{\mathbb{R}^2} f(x,y)\,dA = 1$.

(c) The expected value of X is $\mu_1 = \iint_{\mathbb{R}^2} xf(x,y)\,dA$; the expected value of Y is $\mu_2 = \iint_{\mathbb{R}^2} yf(x,y)\,dA$.

6. (a) $A(S) = \iint_D |\mathbf{r}_u \times \mathbf{r}_v|\,dA$

(b) $$A(S) = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

(c) $A(S) = 2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$

7. (a) $\iiint_B f(x,y,z)\,dV = \lim\limits_{l,m,n\to\infty} \sum\limits_{i=1}^{l}\sum\limits_{j=1}^{m}\sum\limits_{k=1}^{n} f\left(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*\right)\Delta V$

(b) We usually evaluate $\iiint_B f(x,y,z)\,dV$ as an iterated integral according to Fubini's Theorem for Triple Integrals (see Theorem 12.7.4).

(c) See the paragraph following Example 12.7.1.

(d) See (5) and (6) and the accompanying discussion in Section 12.7.

(e) See (10) and the accompanying discussion in Section 12.7.

(f) See (11) and the preceding discussion in Section 12.7.

8. (a) $m = \iiint_E \rho(x,y,z)\,dV$

(b) $M_{yz} = \iiint_E x\rho(x,y,z)\,dV$, $M_{xz} = \iiint_E y\rho(x,y,z)\,dV$, $M_{xy} = \iiint_E z\rho(x,y,z)\,dV$.

(c) The center of mass is $(\overline{x},\overline{y},\overline{z})$ where $\overline{x} = \dfrac{M_{yz}}{m}$, $\overline{y} = \dfrac{M_{xz}}{m}$, and $\overline{z} = \dfrac{M_{xy}}{m}$.

(d) $I_x = \iiint_E (y^2+z^2)\rho(x,y,z)\,dV$, $I_y = \iiint_E (x^2+z^2)\rho(x,y,z)\,dV$,
$I_z = \iiint_E (x^2+y^2)\rho(x,y,z)\,dV$.

9. (a) See Formula 12.8.2 and the accompanying discussion.

(b) See Formula 12.8.4 and the accompanying discussion.

(c) We may want to change from rectangular to cylindrical or spherical coordinates in a triple integral if the region E of integration is more easily described in cylindrical or spherical coordinates or if the triple integral is easier to evaluate using cylindrical or spherical coordinates.

10. (a) $$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \partial x/\partial u & \partial x/\partial v \\ \partial y/\partial u & \partial y/\partial v \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}$$

(b) See (9) and the accompanying discussion in Section 12.9.

(c) See (13) and the accompanying discussion in Section 12.9.