Last Name: ______________________________

First Name: ______________________________

Math 21a Final Exam: Thursday, May 25, 2000
(Regular and Physics Sections)

SECTION (CIRCLE ONE):

Robert Winters John Friedman (CA) MWF 10-11	Marty Weissman Lukasz Fidkowski (CA) MWF 11-12	Tom Weston Nick Stang (CA) MWF 11-12	Russell Mann Mike Grobis (CA) MWF 11-12 (Physics)
Tom Weston James Griffin (CA) MWF 12-1 (BioChem)	Yang Liu Bartok Czech (CA) TTh 10-11:30	Daniel Allcock Nikhil Dutta (CA) TTh 11:30-1	

Question	Points	Score
1	8	
2	8	
3	10	
4	8	
5	8	
6	8	
7	10	
8	6	
9	8	
10	8	
11	10	
12	8	
Total	100	

The time allotted for this exam is <u>3 hours</u>.

Justify your answers carefully. No partial credit can be given for unsubstantiated answers.

If more space is needed, use the back of the previous page and make note of this.

Please write neatly. Answers which are deemed illegible by the grader will not receive credit.

No calculators, computers or other electronic aids are allowed; nor are you allowed to refer to any written notes or source material; nor are you allowed to communicate with other students.

In agreeing to take this exam, you are implicitly accepting Harvard University's Honor Code.

(1) Find all critical points of the function $f(x, y) = x^3 + y^2 - 6xy + 6x + 3y - 2$ and for each point, determine whether it is a local maximum, a local minimum, or a saddle point.

(2) Find the maximum and minimum values of the function $f(x, y) = x - y + 3z$ on the surface given by the equation $x^2 + 2y^2 + 2z^2 = 24$.

(3) Two planes are given by the equations $2x + y + z = 2$ and $x - y - 3z = 4$. Find the point on their line of intersection that is closest to the origin.

(4) Given the line $(x, y, z) = (5 - 2t, 2 + t, -3 + 2t)$ in $\mathbf{R}^3$ and the point $(9, 5, -2)$,

a) find an equation for the plane containing both the line and the point.

b) find the shortest distance from the point $(6, 1, 0)$ to the plane you just found.

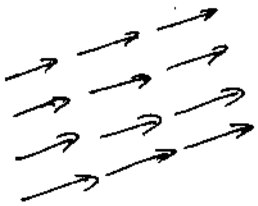
(5) What is the average value of the function $f(x, y) = x^2$ over the unit disk in $\mathbf{R}^2$, i.e., the region $x^2 + y^2 \leq 1$?

(6) What is the directional derivative of the function $f(x, y, z) = x^3 + y^2 + z$ in the direction perpendicular to the surface $xy + 5yz - 3z^2 = 5$ at the point (-1,2,1)?
(Choose the direction with positive z-component.)

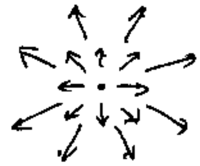
(7) Find the volume of the solid bounded below by $z = 0$, above by $z^2 = x^2 + y^2$, and on the side by $x^2 + y^2 + z^2 = 2$.

(8) Which of the following vector fields are not conservative? Briefly explain your reasoning.

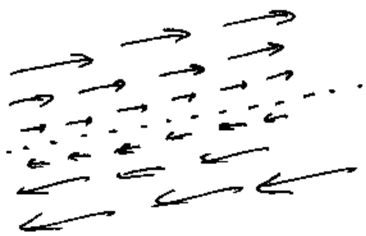
(a)



(b)



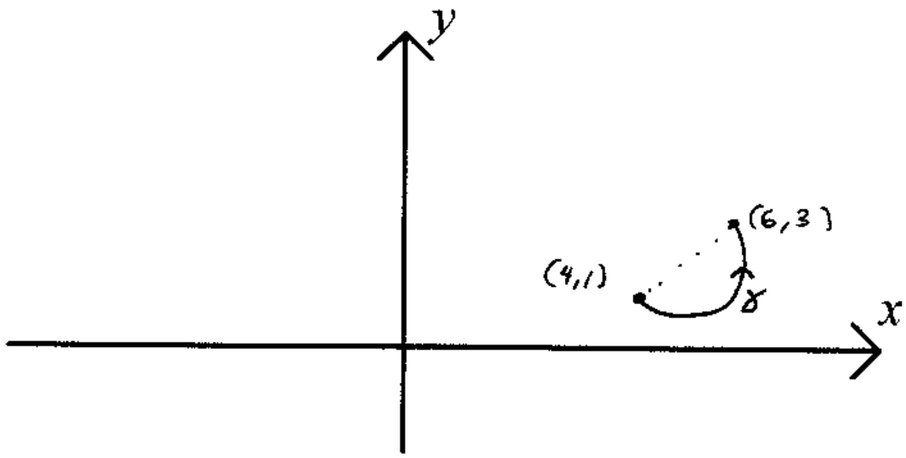
(c)



(d)

(9) Water is flowing down a vertical cylindrical pipe of radius 2 inches. The velocity vector field of the water is given by $\mathbf{v} = (r^2 - 4)\mathbf{k}$ where r is the distance in inches from the center of the pipe.
How much water flows out of the bottom of the pipe in 3 seconds?

(10) Calculate the work integral $\int_\gamma \mathbf{F}\cdot d\mathbf{x}$ for the curve γ shown, a semicircle from the point (4,1) to the point (6,3) where $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (3x - 2y)\mathbf{j}$.
(Hint: It's possible to find the value of this integral without parametrizing the semicircle.)



(11) Compute the flux of the vector field $F(x,y,z) = (e^{y^2+z^2}, y^2+z^2, e^{x^2+y^2})$ across a portion of the cone with equation $4(x^2+y^2) = 9z^2$ lying between $z = 0$ and $z = 2$ oriented with a downward normal.

(12) Let **F** be the vector field given by $\mathbf{F}(x, y, z) = (y^2x, y^2x, y^2x)$. Let D be the portion of the solid ball $x^2 + y^2 + z^2 \leq 9$ which lies in the first octant (i.e. $x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq 9$). Set up, **but do not evaluate,** a triple integral <u>in spherical coordinates</u> which gives the flux of **F** out of the boundary of the region D.