Math 21a Hourly 2 Answers

1. a) $\nabla f = (3, 4)$.
    b) The tangent plane is given by $z - 3x - 4y = -5$.
    c) Using the linear approximation, $f(1.02, 2.05) = f(1, 2) + \nabla f|_{(1,2)} \cdot (.02, .05) = 6.26$.

2. The stationary points occur at $(e^{-a}, e^{-b}, e^{-c})/(e^{-a} + e^{-b} + e^{-c})$.

3. 8/3.

4. a) The stationary points are $(0, 1)$, $(0, -1)$, $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$.
    b) The local maximum is $(0, -1)$. The local minima are $(1, 1)$, $(-1, 1)$. The remaining three are saddles.
    c) If the level set is tangent to the y axis, then ∇f is orthogonal to $(0, 1)$ and so $f_y = 0$. This occurs where $y = \pm 1$. Where $y = 1$, $f = x^4 - 2x^2 - 6$ and so $f = 2$ if $x = \pm 2$. Where $y = -1$, $f = x^4 - 2x^2 + 6$ and so f is not equal to 2 for any value of x. Thus, the points are $(2, 1)$ and $(-2, 1)$.

5. Change the order of integration to write this integral as $\int_0^{\pi/2} \left(\int_{\sin(y)}^1 dx \right) dy = \pi/2 - 1$.

6. Let z denote height, x denote length and y denote width. The you are asked to minimize the function $f(x, y, z) = 50xy + 20(xz + yz)$ where x, y and z are constrained by the requirement $xyz = 20,000,000$. The minimum has $z = 500$ centimeters and $x = y = 200$ centimeters.