Math 21a Spring '98 Exam 2 Solutions

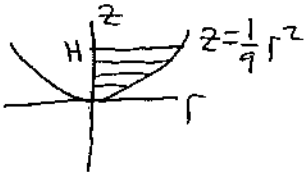
(1) a) $\int_0^1 \int_{-\sqrt{1-x^2}}^{x} 1200x \, dy \, dx = \int_0^1 1200x^2 + 1200x\sqrt{1-x^2} \, dx$

$= 400x^3 - 400(1-x^2)^{3/2}\Big|_{x=0}^{1} = 800$

b) $\int_0^1 \int_0^x (2y)(1200x) \, dy \, dx + \int_0^1 \int_{-\sqrt{1-x^2}}^{0} (-2y)(1200x) \, dy \, dx$

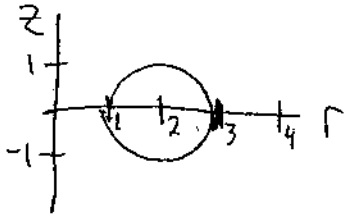
$= \int_0^1 1200x^3 \, dx + \int_0^1 (1-x^2)(1200x) \, dx = 600x^2\Big|_{x=0}^{1} = 600$

(2) $z = \frac{1}{9}(x^2+y^2)$; top area $= 36\cdot\pi$; $\int_0^{2\pi} \int_0^H \int_0^{\sqrt{9z}} r \, dr \, dz \, d\theta = 36\pi$



$36\pi = \frac{9\pi H^2}{2} \Longrightarrow H = 2\sqrt{2}$ inches

(3)



$$\text{vol} = \int_0^{2\pi} \int_{-1}^{1} \int_{2-\sqrt{1-z^2}}^{2+\sqrt{1-z^2}} r \, dr \, dz \, d\theta$$

There are other ways of setting up.

(4)

	$\cos\phi$	$\sin\theta$	$\rho\sin\phi\cos\theta$
half-ball	+	0	0
cone	−	0	0
cube	+	+	+
half-cylinder	0	0	+

(5) a) $x = 1+2t$, $y = 2-6t$, $z = 0+1t$

$\vec{F} = (2-6t)\vec{i} - (1+2t)\vec{j} + (1+2t)(0+1t)\vec{k}$

$d\vec{r} = 2\vec{i} - 6\vec{j} + 1\vec{k}$

$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \left[2(2-6t) + 6(1+2t) + 1(1+2t)t\right] dt = \int_0^1 10 + t + 2t^2 \, dt = \frac{67}{6}$

b) Potential Function $= \frac{x^3}{3} + g(y)$ where $g'(y) = e^{y^2}$ $\Longrightarrow$ $\int_C \vec{F}\cdot d\vec{r} = \frac{x^3}{3} + g(y)\Big|_{(-2,0)}^{(2,0)} = \frac{16}{3}$