

4. $\lim_{t \rightarrow -\infty} \arctan t = \frac{\pi}{2}$, $\lim_{t \rightarrow -\infty} e^{-2t} = 0$, $\lim_{t \rightarrow -\infty} \frac{\ln t}{t} = \lim_{t \rightarrow -\infty} \frac{1/t}{1} = 0$ (by l'Hospital's Rule). Thus

$$\lim_{t \rightarrow -\infty} \left\langle \arctan t, e^{-2t}, \frac{\ln t}{t} \right\rangle = \left\langle \frac{\pi}{2}, 0, 0 \right\rangle.$$

5. $x = \cos 4t$, $y = t$, $z = \sin 4t$. At any point (x, y, z) on the curve, $x^2 + z^2 = \cos^2 4t + \sin^2 4t = 1$. So the curve lies on a circular cylinder with axis the y -axis. Since $y = t$, this is a helix. So the graph is VI.

6. $x = t$, $y = t^2$, $z = e^{-t}$. At any point on the curve, $y = x^2$. So the curve lies on the parabolic cylinder $y = x^2$. Note that y and z are positive for all t , and the point $(0, 0, 1)$ is on the curve (when $t = 0$). As $t \rightarrow \infty$, $(x, y, z) \rightarrow (\infty, \infty, 0)$, while as $t \rightarrow -\infty$, $(x, y, z) \rightarrow (-\infty, \infty, \infty)$, so the graph must be II.

7. $x = t$, $y = 1/(1+t^2)$, $z = t^2$. Note that y and z are positive for all t . The curve passes through $(0, 1, 0)$ when $t = 0$. As $t \rightarrow \infty$, $(x, y, z) \rightarrow (\infty, 0, \infty)$, and as $t \rightarrow -\infty$, $(x, y, z) \rightarrow (-\infty, 0, \infty)$. So the graph is IV.

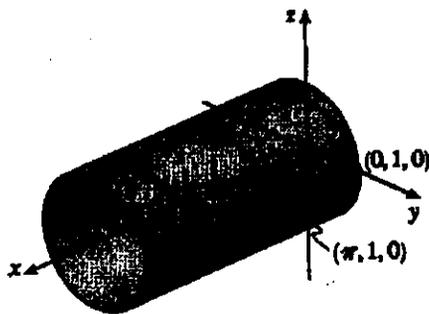
8. $x = e^{-t} \cos 10t$, $y = e^{-t} \sin 10t$, $z = e^{-t}$. $x^2 + y^2 = e^{-2t} \cos^2 10t + e^{-2t} \sin^2 10t = e^{-2t} (\cos^2 10t + \sin^2 10t) = e^{-2t} = z^2$, so the curve lies on the cone $x^2 + y^2 = z^2$. Also, z is always positive; the graph must be I.

9. $x = \cos t$, $y = \sin t$, $z = \sin 5t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z -axis. Each of x , y and z is periodic, and at $t = 0$ and $t = 2\pi$ the curve passes through the same point, so the curve repeats itself and the graph is V.

10. $x = \cos t$, $y = \sin t$, $z = \ln t$. $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$, so the curve lies on a circular cylinder with axis the z -axis. As $t \rightarrow 0$, $z \rightarrow -\infty$, so the graph is III.

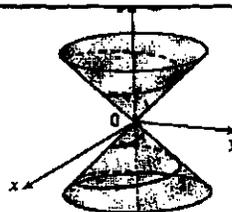
13. The corresponding parametric equations are $x = t$, $y = \cos 2t$, $z = \sin 2t$. Note that

$y^2 + z^2 = \cos^2 2t + \sin^2 2t = 1$, so the curve lies on the circular cylinder $y^2 + z^2 = 1$. Since $x = t$, the curve is a helix.

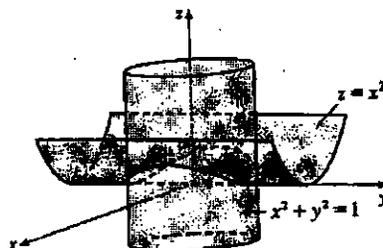


19. If $x = t \cos t$, $y = t \sin t$, and $z = t$, then

$x^2 + y^2 = t^2 \cos^2 t + t^2 \sin^2 t = t^2 = z^2$, so the curve lies on the cone $z^2 = x^2 + y^2$. Since $z = t$, the curve is a spiral on this cone.



20. Here $x^2 = \sin^2 t = z$ and $x^2 + y^2 = \sin^2 t + \cos^2 t = 1$, so the curve is the intersection of the parabolic cylinder $z = x^2$ with the circular cylinder $x^2 + y^2 = 1$.



28. The projection of the curve C of intersection onto the xy -plane is the circle $x^2 + y^2 = 4$, $z = 0$. Then we can write $x = 2 \cos t$, $y = 2 \sin t$, $0 \leq t \leq 2\pi$. Since C also lies on the surface $z = xy$, we have $z = xy = (2 \cos t)(2 \sin t) = 4 \cos t \sin t$, or $2 \sin(2t)$. Then parametric equations for C are $x = 2 \cos t$, $y = 2 \sin t$, $z = 2 \sin(2t)$, $0 \leq t \leq 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + 2 \sin(2t) \mathbf{k}$, $0 \leq t \leq 2\pi$.