6. (a) The graph of f decreases if we start at $(-1,2)$ and move in the positive x-direction, so $f_x(-1,2)$ is negative.

(b) The graph of f decreases if we start at $(-1,2)$ and move in the positive y-direction, so $f_y(-1,2)$ is negative.

(c) $f_{xx} = \frac{\partial}{\partial x}(f_x)$, so f_{xx} is the rate of change of f_x in the x-direction. f_x is negative at $(-1,2)$ and if we move in the positive x-direction, the surface becomes less steep. Thus the values of f_x are increasing and $f_{xx}(-1,2)$ is positive.

(d) f_{yy} is the rate of change of f_y in the y-direction. f_y is negative at $(-1,2)$ and if we move in the positive y-direction, the surface becomes steeper. Thus the values of f_y are decreasing, and $f_{yy}(-1,2)$ is negative.

8. $f_x(2,1)$ is the rate of change of f at $(2,1)$ in the x-direction. If we start at $(2,1)$, where $f(2,1)=10$, and move in the positive x-direction, we reach the next contour line (where $f(x,y)=12$) after approximately 0.6 units. This represents an average rate of change of about $\frac{2}{0.6}$. If we approach the point $(2,1)$ from the left (moving in the positive x-direction) the output values increase from 8 to 10 with an increase in x of approximately 0.9 units, corresponding to an average rate of change of $\frac{2}{0.9}$. A good estimate for $f_x(2,1)$ would be the average of these two, so $f_x(2,1)\approx 2.8$. Similarly, $f_y(2,1)$ is the rate of change of f at $(2,1)$ in the y-direction. If we approach $(2,1)$ from below, the output values decrease from 12 to 10 with a change in y of approximately 1 unit, corresponding to an average rate of change of -2. If we start at $(2,1)$ and move in the positive y-direction, the output values decrease from 10 to 8 after approximately 0.9 units, a rate of change of $\frac{-2}{0.9}$. Averaging these two results, we estimate $f_y(2,1)\approx -2.1$.

18. $f(x,y)=x^y \quad\Rightarrow\quad f_x(x,y)=yx^{y-1},\ f_y(x,y)=x^y\ln x$

26. $f(x,y,z)=x^2e^{yz} \quad\Rightarrow\quad f_x(x,y,z)=2xe^{yz},\ f_y(x,y,z)=x^2e^{yz}(z)=x^2ze^{yz},$
$f_z(x,y,z)=x^2e^{yz}(y)=x^2ye^{yz}.$

48. $f(x,y)=\ln(3x+5y) \quad\Rightarrow\quad f_x(x,y)=\dfrac{3}{3x+5y},\ f_y(x,y)=\dfrac{5}{3x+5y}$. Then
$f_{xx}(x,y)=3(-1)(3x+5y)^{-2}(3)=-\dfrac{9}{(3x+5y)^2},\ f_{xy}(x,y)=-\dfrac{15}{(3x+5y)^2},\ f_{yx}(x,y)=-\dfrac{15}{(3x+5y)^2},$
and $f_{yy}(x,y)=-\dfrac{25}{(3x+5y)^2}.$

52. $u=xye^y \quad\Rightarrow\quad u_x=ye^y,\ u_{xy}=ye^y+e^y=(y+1)e^y$ and $u_y=x(ye^y+e^y)=x(y+1)e^y,$
$u_{yx}=(y+1)e^y$. Thus $u_{xy}=u_{yx}$.