7. $z = e^r \cos\theta,\ r = st,\ \theta = \sqrt{s^2+t^2} \quad \Rightarrow$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial s} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial s} = e^r \cos\theta \cdot t + e^r(-\sin\theta)\cdot \tfrac{1}{2}\left(s^2+t^2\right)^{-1/2}(2s)$$

$$= te^r\cos\theta - e^r\sin\theta\cdot\frac{s}{\sqrt{s^2+t^2}} = e^r\left(t\cos\theta - \frac{s}{\sqrt{s^2+t^2}}\sin\theta\right)$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial z}{\partial \theta}\frac{\partial \theta}{\partial t} = e^r \cos\theta \cdot s + e^r(-\sin\theta)\cdot \tfrac{1}{2}\left(s^2+t^2\right)^{-1/2}(2t)$$

$$= se^r\cos\theta - e^r\sin\theta\cdot\frac{t}{\sqrt{s^2+t^2}} = e^r\left(s\cos\theta - \frac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$$

10. By the Chain Rule (3), $\dfrac{\partial W}{\partial s} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial s} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial s}$. Then

$$W_s(1,0) = F_u\left(u(1,0), v(1,0)\right)u_s(1,0) + F_v\left(u(1,0), v(1,0)\right)v_s(1,0)$$
$$= F_u(2,3)u_s(1,0) + F_v(2,3)v_s(1,0) = (-1)(-2) + (10)(5) = 52$$

Similarly, $\dfrac{\partial W}{\partial t} = \dfrac{\partial W}{\partial u}\dfrac{\partial u}{\partial t} + \dfrac{\partial W}{\partial v}\dfrac{\partial v}{\partial t} \quad \Rightarrow$

$$W_t(1,0) = F_u\left(u(1,0), v(1,0)\right)u_t(1,0) + F_v\left(u(1,0), v(1,0)\right)v_t(1,0)$$
$$= F_u(2,3)u_t(1,0) + F_v(2,3)v_t(1,0) = (-1)(6) + (10)(4) = 34$$

17. $z = y^2\tan x,\ x = t^2uv,\ y = u + tv^2 \quad \Rightarrow$

$\partial z/\partial t = \left(y^2\sec^2 x\right)2tuv + (2y\tan x)\,v^2$, $\partial z/\partial u = \left(y^2\sec^2 x\right)t^2v + 2y\tan x$,
$\partial z/\partial v = \left(y^2\sec^2 x\right)t^2u + (2y\tan x)\,2tv$. When $t = 2$, $u = 1$ and $v = 0$, we have $x = 0$, $y = 1$, so $\partial z/\partial t = 0$, $\partial z/\partial u = 0$, $\partial z/\partial v = 4$.

18. $z = \dfrac{x}{y},\ x = re^{st},\ y = rse^t \quad \Rightarrow$

$\dfrac{\partial z}{\partial r} = \dfrac{1}{y}e^{st} + \dfrac{-x}{y^2}se^t$, $\dfrac{\partial z}{\partial s} = \dfrac{1}{y}rte^{st} - \dfrac{x}{y^2}re^t$, $\dfrac{\partial z}{\partial t} = \dfrac{1}{y}rse^{st} - \dfrac{x}{y^2}rse^t$. When $r = 1$, $s = 2$ and $t = 0$, we have $x = 1$, $y = 2$, so $\partial z/\partial r = \frac{1}{2} + \frac{-1}{4}\cdot 2 = 0$, $\partial z/\partial s = 0 - \frac{1}{4} = -\frac{1}{4}$ and $\partial z/\partial t = \frac{1}{2}\cdot 2 - \frac{1}{4}\cdot 2 = \frac{1}{2}$.

28. (a) Since $\partial W/\partial T$ is negative, a rise in average temperature (while annual rainfall remains constant) causes a decrease in wheat production at the current production levels. Since $\partial W/\partial R$ is positive, an increase in annual rainfall (while the average temperature remains constant) causes an increase in wheat production.

(b) Since the average temperature is rising at a rate of 0.15 °C/year, we know that $dT/dt = 0.15$. Since rainfall is decreasing at a rate of 0.1 cm/year, we know $dR/dt = -0.1$. Then, by the Chain Rule,
$\dfrac{dW}{dt} = \dfrac{\partial W}{\partial T}\dfrac{dT}{dt} + \dfrac{\partial W}{\partial R}\dfrac{dR}{dt} = (-2)(0.15) + (8)(-0.1) = -1.1$. Thus we estimate that wheat production will decrease at a rate of 1.1 units/year.