

1. First we draw a line passing through Raleigh and the eye of the hurricane. We can approximate the directional derivative at Raleigh in the direction of the eye of the hurricane by the average rate of change of pressure between the points where this line intersects the contour lines closest to Raleigh. In the direction of the eye of the hurricane, the pressure changes from 996 millibars to 992 millibars. We estimate the distance between these two points to be approximately 40 miles, so the rate of change of pressure in the direction given is approximately
- $$\frac{992 - 996}{40} = -0.1 \text{ millibar/mi.}$$

8.  $f(x, y) = y \ln x$

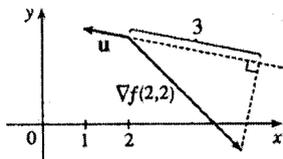
(a)  $\nabla f(x, y) = \langle f_x(x, y), f_y(x, y) \rangle = \langle y/x, \ln x \rangle$       (b)  $\nabla f(1, -3) = \langle \frac{-3}{1}, \ln 1 \rangle = \langle -3, 0 \rangle$

(c) By Equation 9,  $D_{\mathbf{u}}f(1, -3) = \nabla f(1, -3) \cdot \mathbf{u} = \langle -3, 0 \rangle \cdot \langle -\frac{4}{5}, \frac{3}{5} \rangle = \frac{12}{5}$ .

11.  $f(x, y) = 1 + 2x\sqrt{y} \Rightarrow \nabla f(x, y) = \langle 2\sqrt{y}, 2x \cdot \frac{1}{2}y^{-1/2} \rangle = \langle 2\sqrt{y}, x/\sqrt{y} \rangle$ ,  $\nabla f(3, 4) = \langle 4, \frac{3}{2} \rangle$ , and a unit vector in the direction of  $\mathbf{v}$  is  $\mathbf{u} = \frac{1}{\sqrt{4^2 + (-3)^2}} \langle 4, -3 \rangle = \langle \frac{4}{5}, -\frac{3}{5} \rangle$ , so

$$D_{\mathbf{u}}f(3, 4) = \nabla f(3, 4) \cdot \mathbf{u} = \langle 4, \frac{3}{2} \rangle \cdot \langle \frac{4}{5}, -\frac{3}{5} \rangle = \frac{23}{10}.$$

16.  $D_{\mathbf{u}}f(2, 2) = \nabla f(2, 2) \cdot \mathbf{u}$ , the scalar projection of  $\nabla f(2, 2)$  onto  $\mathbf{u}$ , so we draw a perpendicular from the tip of  $\nabla f(2, 2)$  to the line containing  $\mathbf{u}$ . We can use the point  $(2, 2)$  to determine the scale of the axes, and we estimate the length of the projection to be approximately 3.0 units. Since the angle between  $\nabla f(2, 2)$  and  $\mathbf{u}$  is greater than  $90^\circ$ , the scalar projection is negative. Thus  $D_{\mathbf{u}}f(2, 2) \approx -3$ .



change is  $|\nabla f(1, 0)| = 2$  in the direction  $\langle 0, 1 \rangle$ .

20.  $f(x, y) = \ln(x^2 + y^2) \Rightarrow \nabla f(x, y) = \left\langle \frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2} \right\rangle$ ,  $\nabla f(1, 2) = \left\langle \frac{2}{5}, \frac{4}{5} \right\rangle$ . Thus the maximum rate of change is  $|\nabla f(1, 2)| = \frac{2\sqrt{5}}{5}$  in the direction  $\langle \frac{2}{5}, \frac{4}{5} \rangle$  or  $\langle 2, 4 \rangle$ .

26. The fisherman is traveling in the direction  $\langle -80, -60 \rangle$ . A unit vector in this direction is  $\mathbf{u} = \frac{1}{100} \langle -80, -60 \rangle = \langle -\frac{4}{5}, -\frac{3}{5} \rangle$ , and if the depth of the lake is given by  $f(x, y) = 200 + 0.02x^2 - 0.001y^3$ , then  $\nabla f(x, y) = \langle 0.04x, -0.003y^2 \rangle$ .  $D_{\mathbf{u}}f(80, 60) = \nabla f(80, 60) \cdot \mathbf{u} = \langle 3.2, -10.8 \rangle \cdot \langle -\frac{4}{5}, -\frac{3}{5} \rangle = 3.92$ . Since  $D_{\mathbf{u}}f(80, 60)$  is positive, the depth of the lake is increasing near  $(80, 60)$  in the direction toward the buoy.

33. (a)  $\nabla(au + bv) = \left\langle \frac{\partial(au + bv)}{\partial x}, \frac{\partial(au + bv)}{\partial y} \right\rangle = \left\langle a \frac{\partial u}{\partial x} + b \frac{\partial v}{\partial x}, a \frac{\partial u}{\partial y} + b \frac{\partial v}{\partial y} \right\rangle$

$$= a \left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right\rangle + b \left\langle \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \right\rangle = a \nabla u + b \nabla v$$

(b)  $\nabla(uv) = \left\langle v \frac{\partial u}{\partial x} + u \frac{\partial v}{\partial x}, v \frac{\partial u}{\partial y} + u \frac{\partial v}{\partial y} \right\rangle = v \left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right\rangle + u \left\langle \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \right\rangle = v \nabla u + u \nabla v$

44. Since  $\nabla f(x_0, y_0, z_0) = \langle 2x_0, 4y_0, 6z_0 \rangle$  and  $\langle 3, -1, 3 \rangle$  are both normal vectors to the surface at  $(x_0, y_0, z_0)$ , we need  $\langle 2x_0, 4y_0, 6z_0 \rangle = c \langle 3, -1, 3 \rangle$  or  $\langle x_0, 2y_0, 3z_0 \rangle = k \langle 3, -1, 3 \rangle$ . Thus  $x_0 = 3k$ ,  $y_0 = -\frac{1}{2}k$  and  $z_0 = k$ . But  $x_0^2 + 2y_0^2 + 3z_0^2 = 1$  or  $(9 + \frac{1}{2} + 3)k^2 = 1$ , so  $k = \pm \frac{\sqrt{2}}{5}$  and there are two such points:  $(\pm \frac{3\sqrt{2}}{5}, \mp \frac{1}{5\sqrt{2}}, \pm \frac{\sqrt{2}}{5})$ .

47. Let  $(x_0, y_0, z_0)$  be a point on the surface. Then an equation of the tangent plane at the point is

$$\frac{x}{2\sqrt{x_0}} + \frac{y}{2\sqrt{y_0}} + \frac{z}{2\sqrt{z_0}} = \frac{\sqrt{x_0} + \sqrt{y_0} + \sqrt{z_0}}{2}. \text{ But } \sqrt{x_0} + \sqrt{y_0} + \sqrt{z_0} = \sqrt{c}, \text{ so the equation is}$$

$$\frac{x}{\sqrt{x_0}} + \frac{y}{\sqrt{y_0}} + \frac{z}{\sqrt{z_0}} = \sqrt{c}. \text{ The } x\text{-, } y\text{-, and } z\text{-intercepts are } \sqrt{cx_0}, \sqrt{cy_0} \text{ and } \sqrt{cz_0} \text{ respectively. (The } x\text{-intercept}$$

is found by setting  $y = z = 0$  and solving the resulting equation for  $x$ , and the  $y$ - and  $z$ -intercepts are found similarly.) So the sum of the intercepts is  $\sqrt{c}(\sqrt{x_0} + \sqrt{y_0} + \sqrt{z_0}) = c$ , a constant.

48. Let  $(x_0, y_0, z_0)$  be a point on the sphere. Then the normal line is given by  $\frac{x - x_0}{2x_0} = \frac{y - y_0}{2y_0} = \frac{z - z_0}{2z_0}$ . For the

center  $(0, 0, 0)$  to be on the line, we need  $-\frac{x_0}{2x_0} = -\frac{y_0}{2y_0} = -\frac{z_0}{2z_0}$  or equivalently  $1 = 1 = 1$ , which is true.