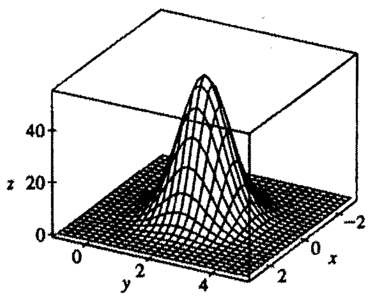
4. In the figure, points at approximately $(-1,1)$ and $(-1,-1)$ are enclosed by oval-shaped level curves which indicate that as we move away from either point in any direction, the values of f are increasing. Hence we would expect local minima at or near $(-1,\pm1)$. Similarly, the point $(1,0)$ appears to be enclosed by oval-shaped level curves which indicate that as we move away from the point in any direction the values of f are decreasing, so we should have a local maximum there. We also show hyperbola-shaped level curves near the points $(-1,0)$, $(1,1)$, and $(1,-1)$. The values of f increase along some paths leaving these points and decrease in others, so we should have a saddle point at each of these points.

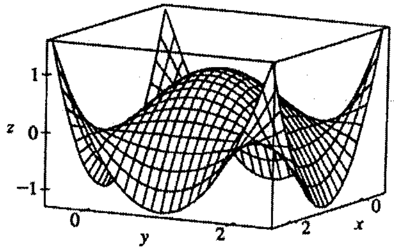
To confirm our predictions, we have $f(x,y) = 3x - x^3 - 2y^2 + y^4 \Rightarrow f_x(x,y) = 3 - 3x^2$, $f_y(x,y) = -4y + 4y^3$. Setting these partial derivatives equal to 0, we have $3 - 3x^2 = 0 \Rightarrow x = \pm1$ and $-4y + 4y^3 = 0 \Rightarrow y(y^2-1) = 0 \Rightarrow y = 0, \pm1$. So our critical points are $(\pm1, 0)$, $(\pm1,\pm1)$. The second partial derivatives are $f_{xx}(x,y) = -6x$, $f_{xy}(x,y) = 0$, and $f_{yy}(x,y) = 12y^2 - 4$, so $D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - [f_{xy}(x,y)]^2 = (-6x)(12y^2-4) - (0)^2 = -72xy^2 + 24x$. We use the Second Derivatives Test to classify the 6 critical points:

Critical Point	D	f_{xx}	Conclusion
$(1,0)$	24	-6	$D > 0,\ f_{xx} < 0 \Rightarrow f$ has a local maximum at $(1,0)$
$(1,1)$	-48		$D < 0 \Rightarrow f$ has a saddle point at $(1,1)$
$(1,-1)$	-48		$D < 0 \Rightarrow f$ has a saddle point at $(1,-1)$
$(-1,0)$	-24		$D < 0 \Rightarrow f$ has a saddle point at $(-1,0)$
$(-1,1)$	48	6	$D > 0,\ f_{xx} > 0 \Rightarrow f$ has a local minimum at $(-1,1)$
$(-1,-1)$	48	6	$D > 0,\ f_{xx} > 0 \Rightarrow f$ has a local minimum at $(-1,-1)$

8. $f(x,y) = e^{4y-x^2-y^2} \Rightarrow f_x = -2xe^{4y-x^2-y^2}$, $f_y = (4-2y)e^{4y-x^2-y^2}$, $f_{xx} = (4x^2-2)e^{4y-x^2-y^2}$, $f_{xy} = -2x(4-2y)e^{4y-x^2-y^2}$, $f_{yy} = (4y^2 - 16y + 14)e^{4y-x^2-y^2}$. Then $f_x = 0$ and $f_y = 0$ implies $x = 0$ and $y = 2$, so the only critical point is $(0,2)$. $D(0,2) = (-2e^4)(-2e^4) - 0^2 = 4e^8 > 0$ and $f_{xx}(0,2) = -2e^4 < 0$, so $f(0,2) = e^4$ is a local maximum.

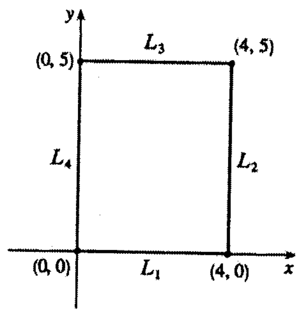


14. $f(x,y) = (2x - x^2)(2y - y^2) \Rightarrow f_x = (2-2x)(2y-y^2)$, $f_y = (2x-x^2)(2-2y)$, $f_{xx} = -2(2y-y^2)$, $f_{yy} = -2(2x-x^2)$ and $f_{xy} = (2-2x)(2-2y)$. Then $f_x = 0$ implies $x = 1$ or $y = 0$ or $y = 2$ and when $x = 1$, $f_y = 0$ implies $y = 1$, when $y = 0$, $f_y = 0$ implies $x = 0$ or $x = 2$ and when $y = 2$, $f_y = 0$ implies $x = 0$ or $x = 2$. Thus the critical points are $(1,1)$, $(0,0)$, $(2,0)$, $(0,2)$ and $(2,2)$.



Now $D(0,0) = D(2,0) = D(0,2) = D(2,2) = -16$ so these critical points are saddle points, and $D(1,1) = 4$ with $f_{xx}(1,1) = -2$, so $f(1,1) = 1$ is a local maximum.

26. $f_x(x,y) = 4 - 2x$ and $f_y(x,y) = 6 - 2y$, so the only critical point is $(2,3)$ (which is in D) where $f(2,3) = 13$. Along L_1: $y = 0$, so $f(x,0) = 4x - x^2 = -(x-2)^2 + 4$, $0 \le x \le 4$, which has a maximum value when $x = 2$ where $f(2,0) = 4$ and a minimum value both when $x = 0$ and $x = 4$, where $f(0,0) = f(4,0) = 0$. Along L_2: $x = 4$, so $f(4,y) = 6y - y^2 = -(y-3)^2 + 9$, $0 \le y \le 5$, which has a maximum value when $y = 3$ where $f(4,3) = 9$ and a minimum value when $y = 0$ where $f(4,0) = 0$. Along L_3: $y = 5$, so $f(x,5) = -x^2 + 4x + 5 = -(x-2)^2 + 9$, $0 \le x \le 4$, which has a maximum value when $x = 2$ where $f(2,5) = 9$ and a minimum value both when $x = 0$ and $x = 4$, where $f(0,5) = f(4,5) = 5$. Along L_4: $x = 0$, so $f(0,y) = 6y - y^2 = -(y-3)^2 + 9$, $0 \le y \le 5$, which has a maximum value when $y = 3$ where $f(0,3) = 9$ and a minimum value when $y = 0$ where $f(0,0) = 0$. Thus the absolute maximum is $f(2,3) = 13$ and the absolute minimum is attained at both $(0,0)$ and $(4,0)$, where $f(0,0) = f(4,0) = 0$.



35. $x + y + z = 100$, so maximize $f(x,y) = xy(100 - x - y)$. $f_x = 100y - 2xy - y^2$, $f_y = 100x - x^2 - 2xy$, $f_{xx} = -2y$, $f_{yy} = -2x$, $f_{xy} = 100 - 2x - 2y$. Then $f_x = 0$ implies $y = 0$ or $y = 100 - 2x$. Substituting $y = 0$ into $f_y = 0$ gives $x = 0$ or $x = 100$ and substituting $y = 100 - 2x$ into $f_y = 0$ gives $3x^2 - 100x = 0$ so $x = 0$ or $\frac{100}{3}$. Thus the critical points are $(0,0)$, $(100,0)$, $(0,100)$ and $\left(\frac{100}{3}, \frac{100}{3}\right)$.

$D(0,0) = D(100,0) = D(0,100) = -10{,}000$ while $D\left(\frac{100}{3}, \frac{100}{3}\right) = \frac{10{,}000}{3}$ and $f_{xx}\left(\frac{100}{3}, \frac{100}{3}\right) = -\frac{200}{3} < 0$. Thus $(0,0)$, $(100,0)$ and $(0,100)$ are saddle points whereas $f\left(\frac{100}{3}, \frac{100}{3}\right)$ is a local maximum. Thus the numbers are $x = y = z = \frac{100}{3}$.

42. The cost equals $5xy + 2(xz + yz)$ and $xyz = V$, so $C(x,y) = 5xy + 2V(x+y)/(xy) = 5xy + 2V\left(x^{-1} + y^{-1}\right)$. Then $C_x = 5y - 2Vx^{-2}$, $C_y = 5x - 2Vy^{-2}$, $f_x = 0$ implies $y = 2V/(5x^2)$, $f_y = 0$ implies $x = \sqrt[3]{\frac{2}{5}V} = y$. Thus the dimensions of the box which minimize the cost are $x = y = \sqrt[3]{\frac{2}{5}V}$ units, $z = V^{1/3}\left(\frac{5}{2}\right)^{2/3}$.