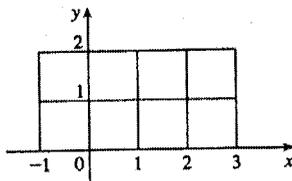


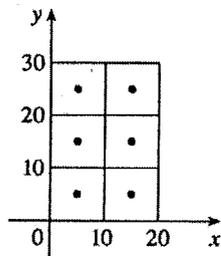
4. The subrectangles are shown in the figure.



Since  $\Delta A = 1$ , we estimate

$$\begin{aligned} \iint_R (y^2 - 2x^2) dA &\approx \sum_{i=1}^4 \sum_{j=1}^2 f(x_{ij}^*, y_{ij}^*) \Delta A \\ &= f(-1, 1) \Delta A + f(-1, 2) \Delta A + f(0, 1) \Delta A + f(0, 2) \Delta A \\ &\quad + f(1, 1) \Delta A + f(1, 2) \Delta A + f(2, 1) \Delta A + f(2, 2) \Delta A \\ &= -1(1) + 2(1) + 1(1) + 4(1) - 1(1) + 2(1) - 7(1) - 4(1) = -4 \end{aligned}$$

6. To approximate the volume, let  $R$  be the planar region corresponding to the surface of the water in the pool, and place  $R$  on coordinate axes so that  $x$  and  $y$  correspond to the dimensions given. Then we define  $f(x, y)$  to be the depth of the water at  $(x, y)$ , so the volume of water in the pool is the volume of the solid that lies above the rectangle  $R = [0, 20] \times [0, 30]$  and below the graph of  $f(x, y)$ . We can estimate this volume using the Midpoint Rule with  $m = 2$  and  $n = 3$ , so  $\Delta A = 100$ .



Each subrectangle with its midpoint is shown in the figure. Then

$$\begin{aligned} V &\approx \sum_{i=1}^2 \sum_{j=1}^3 f(\bar{x}_i, \bar{y}_j) \Delta A \\ &= \Delta A [f(5, 5) + f(5, 15) + f(5, 25) + f(15, 5) + f(15, 15) + f(15, 25)] \\ &= 100(3 + 7 + 10 + 3 + 5 + 8) = 3600 \end{aligned}$$

Thus, we estimate that the pool contains 3600 cubic feet of water.

Alternatively, we can approximate the volume with a Riemann sum where  $m = 4$ ,  $n = 6$  and the sample points are taken to be, for example, the upper right corner of each subrectangle. Then  $\Delta A = 25$  and

$$\begin{aligned} V &\approx \sum_{i=1}^4 \sum_{j=1}^6 f(x_i, y_j) \Delta A \\ &= 25[3 + 4 + 7 + 8 + 10 + 8 + 4 + 6 + 8 + 10 + 12 + 10 + 3 + 4 \\ &\quad + 5 + 6 + 8 + 7 + 2 + 2 + 2 + 3 + 4 + 4] \\ &= 25(140) = 3500 \end{aligned}$$

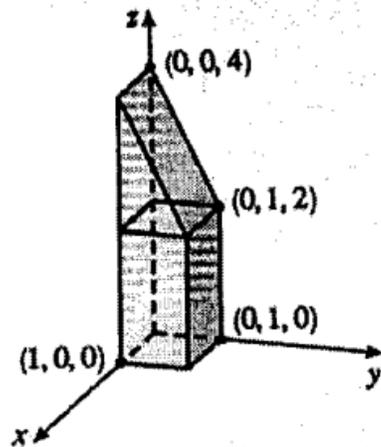
So we estimate that the pool contains 3500 ft<sup>3</sup> of water.

7. The values of  $f(x, y) = \sqrt{52 - x^2 - y^2}$  get smaller as we move farther from the origin, so on any of the subrectangles in the problem, the function will have its largest value at the lower left corner of the subrectangle and its smallest value at the upper right corner, and any other value will lie between these two. So using these subrectangles we have  $U < V < L$ . (Note that this is true no matter how  $R$  is divided into subrectangles.)

8. From the level curves we see that  $f(\frac{1}{2}, \frac{1}{2}) \approx 11$ . So, using the Midpoint Rule with only one subrectangle, we get  $\iint_R f(x, y) dA \approx 1 \cdot f(\frac{1}{2}, \frac{1}{2}) \approx 11$ . Dividing  $R$  into four squares of equal size, we get  $\iint_R f(x, y) dA \approx \frac{1}{4} [f(\frac{1}{4}, \frac{1}{4}) + f(\frac{1}{4}, \frac{3}{4}) + f(\frac{3}{4}, \frac{1}{4}) + f(\frac{3}{4}, \frac{3}{4})] \approx \frac{1}{4} (11 + 13 + 9.5 + 11) \approx 11$ . Using sixteen squares we get the same result. So  $\iint_R f(x, y) dA \approx 11$ .

13.  $z = f(x, y) = 4 - 2y \geq 0$  for  $0 \leq y \leq 1$ . Thus the integral represents the volume of that part of the rectangular solid  $[0, 1] \times [0, 1] \times [0, 4]$  which lies below the plane  $z = 4 - 2y$ . So

$$\iint_R (4 - y) dA = (1)(1)(2) + \frac{1}{2}(1)(1)(2) = 3$$



14. Here  $z = \sqrt{9 - y^2}$ , so  $z^2 + y^2 = 9$ ,  $z \geq 0$ . Thus the integral represents the volume of the top half of the part of the circular cylinder  $z^2 + y^2 = 9$  that lies above the rectangle  $[0, 4] \times [0, 2]$ .

