7. $\int_1^4 \int_1^2 \left(\frac{x}{y}+\frac{y}{x}\right) dy\,dx = \int_1^4 \left[x\ln|y| + \frac{1}{x}\cdot\frac{1}{2}y^2\right]_{y=1}^{y=2} dx = \int_1^4 \left(x\ln 2 + \frac{3}{2x}\right) dx$

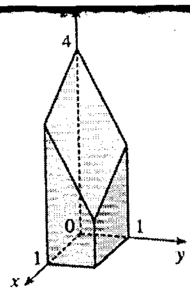
$= \left[\frac{1}{2}x^2\ln 2 + \frac{3}{2}\ln|x|\right]_1^4 = 8\ln 2 + \frac{3}{2}\ln 4 - \frac{1}{2}\ln 2$

$= \frac{15}{2}\ln 2 + 3\ln 4^{1/2} = \frac{21}{2}\ln 2$

13. $\iint_R \frac{xy^2}{x^2+1}\,dA = \int_0^1 \int_{-3}^3 \frac{xy^2}{x^2+1}\,dy\,dx = \int_0^1 \frac{x}{x^2+1}\,dx \int_{-3}^3 y^2\,dy$

$= \left[\frac{1}{2}\ln(x^2+1)\right]_0^1 \left[\frac{1}{3}y^3\right]_{-3}^3 = \frac{1}{2}(\ln 2 - \ln 1)\cdot\frac{1}{3}(27+27) = 9\ln 2$

17. $z = f(x,y) = 4 - x - 2y \geq 0$ for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.
So the solid is the region in the first octant which lies below the plane $z = 4 - x - 2y$ and above $[0,1]\times[0,1]$.



19. $V = \iint_R (2x+5y+1)\,dA = \int_1^4 \int_{-1}^0 (2x+5y+1)\,dx\,dy = \int_1^4 \left[x^2+5xy+x\right]_{x=-1}^{x=0} dy$

$= \int_1^4 5y\,dy = \frac{5}{2}y^2\Big]_1^4 = \frac{75}{2}$

21. $V = \int_{-2}^2 \int_{-1}^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy = 4\int_0^2 \int_0^1 \left(1 - \frac{1}{4}x^2 - \frac{1}{9}y^2\right) dx\,dy$

$= 4\int_0^2 \left[x - \frac{1}{12}x^3 - \frac{1}{9}y^2 x\right]_{x=0}^{x=1} dy = 4\int_0^2 \left(\frac{11}{12} - \frac{1}{9}y^2\right) dy = 4\left[\frac{11}{12}y - \frac{1}{27}y^3\right]_0^2 = 4\cdot\frac{83}{54} = \frac{166}{27}$

23. Here we need the volume of the solid lying under the surface $z = x\sqrt{x^2+y}$ and above the square $R = [0,1]\times[0,1]$ in the xy-plane.

$$V = \int_0^1 \int_0^1 x\sqrt{x^2+y}\,dx\,dy = \int_0^1 \frac{1}{3}\left[(x^2+y)^{3/2}\right]_{x=0}^{x=1} dy = \frac{1}{3}\int_0^1 \left[(1+y)^{3/2} - y^{3/2}\right] dy$$

$$= \frac{1}{3}\cdot\frac{2}{5}\left[(1+y)^{5/2} - y^{5/2}\right]_0^1 = \frac{4}{15}\left(2\sqrt{2}-1\right)$$