5. $\int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} ze^y \, dx \, dz \, dy = \int_0^3 \int_0^1 \left[xze^y \right]_{x=0}^{x=\sqrt{1-z^2}} dz \, dy = \int_0^3 \int_0^1 ze^y \sqrt{1-z^2} \, dz \, dy$

$$= \int_0^3 \left[-\tfrac{1}{3}\left(1-z^2\right)^{3/2} e^y \right]_{z=0}^{z=1} dy = \int_0^3 \tfrac{1}{3} e^y \, dy = \tfrac{1}{3} e^y \Big]_0^3 = \tfrac{1}{3}\left(e^3 - 1\right)$$

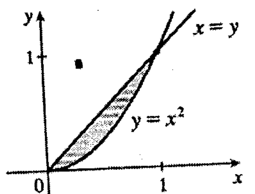
9. Here $E = \{(x,y,z) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}, 0 \le z \le 1 + x + y\}$, so

$$\iiint_E 6xy \, dV = \int_0^1 \int_0^{\sqrt{x}} \int_0^{1+x+y} 6xy \, dz \, dy \, dx = \int_0^1 \int_0^{\sqrt{x}} \left[6xyz \right]_{z=0}^{z=1+x+y} dy \, dx$$

$$= \int_0^1 \int_0^{\sqrt{x}} 6xy(1+x+y) \, dy \, dx = \int_0^1 \left[3xy^2 + 3x^2y^2 + 2xy^3 \right]_{y=0}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left(3x^2 + 3x^3 + 2x^{5/2} \right) dx = \left[x^3 + \tfrac{3}{4}x^4 + \tfrac{4}{7}x^{7/2} \right]_0^1 = \tfrac{65}{28}$$

12.



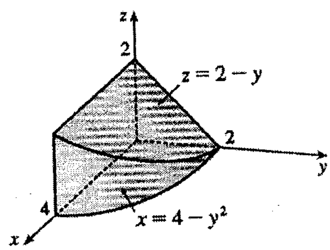
E is the solid above the region shown in the xy-plane and below the plane $z = x$. Thus,

$$\iiint_E (x + 2y) \, dV = \int_0^1 \int_{x^2}^{x} \int_0^x (x + 2y) \, dz \, dy \, dx$$

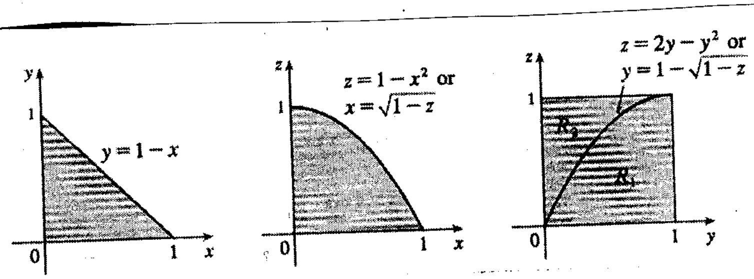
$$= \int_0^1 \int_{x^2}^{x} \left(x^2 + 2yx \right) dy \, dx = \int_0^1 \left[x^2 y + xy^2 \right]_{y=x^2}^{y=x} dx$$

$$= \int_0^1 \left(2x^3 - x^4 - x^5 \right) dx = \left[\tfrac{1}{2}x^4 - \tfrac{1}{5}x^5 - \tfrac{1}{6}x^6 \right]_0^1 = \tfrac{2}{15}$$

24. $E = \{(x,y,z) \mid 0 \le y \le 2, 0 \le z \le 2 - y, 0 \le x \le 4 - y^2\}$, the solid bounded by the three coordinate planes, the plane $z = 2 - y$, and the cylindrical surface $x = 4 - y^2$.



30.



The projections of E onto the xy- and xz-planes are as in the first two diagrams and so

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z) \, dy \, dz \, dx = \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x,y,z) \, dy \, dx \, dz$$

$$= \int_0^1 \int_0^{1-y} \int_0^{1-x^2} f(x,y,z) \, dz \, dx \, dy = \int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z) \, dz \, dy \, dx$$

Now the surface $z = 1 - x^2$ intersects the plane $y = 1 - x$ in a curve whose projection in the yz-plane is $z = 1 - (1-y)^2$ or $z = 2y - y^2$. So we must split up the projection of E on the yz-plane into two regions as in the third diagram. For (y,z) in R_1, $0 \le x \le 1 - y$ and for (y,z) in R_2, $0 \le x \le \sqrt{1-z}$, and so the given integral is also equal to

$$\int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} f(x,y,z) \, dx \, dy \, dz + \int_0^1 \int_{1-\sqrt{1-z}}^{1} \int_0^{1-y} f(x,y,z) \, dx \, dy \, dz$$

$$= \int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f(x,y,z) \, dx \, dz \, dy + \int_0^1 \int_{2y-y^2}^{1} \int_0^{\sqrt{1-z}} f(x,y,z) \, dx \, dz \, dy.$$