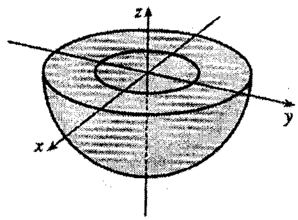
4. The region of integration is given in spherical coordinates by $E = \{(\rho, \theta, \phi) \mid 1 \le \rho \le 2, 0 \le \theta \le 2\pi, \pi/2 \le \phi \le \pi\}$. This represents the solid region between the spheres $\rho = 1$ and $\rho = 2$ and below the xy-plane.



$$\int_0^{2\pi} \int_{\pi/2}^{\pi} \int_1^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \int_0^{2\pi} d\theta \int_{\pi/2}^{\pi} \sin\phi \, d\phi \int_1^2 \rho^2 \, d\rho$$

$$= [\theta]_0^{2\pi} \, [-\cos\phi]_{\pi/2}^{\pi} \, \left[\tfrac{1}{3}\rho^3\right]_1^2$$

$$= 2\pi\,(1)\left(\tfrac{7}{3}\right) = \tfrac{14\pi}{3}$$

11. In cylindrical coordinates, E is bounded by the cylinder $r = 1$, the plane $z = 0$, and the cone $z = 2r$. So $E = \{(r, \theta, z) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1, 0 \le z \le 2r\}$ and

$$\iiint_E x^2 \, dV = \int_0^{2\pi} \int_0^1 \int_0^{2r} r^2 \cos^2\theta \, r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 \left[r^3 \cos^2\theta \, z\right]_{z=0}^{z=2r} dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 2r^4 \cos^2\theta \, dr \, d\theta = \int_0^{2\pi} \left[\tfrac{2}{5} r^5 \cos^2\theta\right]_{r=0}^{r=1} d\theta = \tfrac{2}{5} \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= \frac{2}{5} \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1}{5}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi} = \frac{2\pi}{5}$$

19. $$\iiint_E \sqrt{x^2 + y^2 + z^2} \, dV = \int_0^{2\pi} \int_0^{\pi/6} \int_0^2 (\rho)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\pi/6} \sin\phi \, d\phi \int_0^2 \rho^3 \, d\rho = [\theta]_0^{2\pi} \, [-\cos\phi]_0^{\pi/6} \, \left[\tfrac{1}{4}\rho^4\right]_0^2$$

$$= (2\pi)\left(1 - \tfrac{\sqrt{3}}{2}\right)(4) = 8\pi\left(1 - \tfrac{\sqrt{3}}{2}\right) = 4\pi\left(2 - \sqrt{3}\right)$$

20. In spherical coordinates, the sphere $x^2 + y^2 + z^2 = 4$ is equivalent to $\rho = 2$ and the cone $z = \sqrt{x^2 + y^2}$ is represented by $\phi = \frac{\pi}{4}$. Thus, the solid is given by $\{(\rho, \theta, \phi) \mid 0 \le \rho \le 2, 0 \le \theta \le 2\pi, \frac{\pi}{4} \le \phi \le \frac{\pi}{2}\}$ and $V = \int_{\pi/4}^{\pi/2} \int_0^{2\pi} \int_0^2 \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi = \int_{\pi/4}^{\pi/2} \sin\phi \, d\phi \int_0^{2\pi} d\theta \int_0^2 \rho^2 \, d\rho$

$$= [-\cos\phi]_{\pi/4}^{\pi/2} \, [\theta]_0^{2\pi} \, \left[\tfrac{1}{3}\rho^3\right]_0^2 = \left(\tfrac{\sqrt{2}}{2}\right)(2\pi)\left(\tfrac{8}{3}\right) = \tfrac{8\sqrt{2}\pi}{3}$$

29. The region E of integration is the region above the paraboloid $z = x^2 + y^2$, or $z = r^2$, and below the paraboloid $z = 2 - x^2 - y^2$, or $z = 2 - r^2$. Also, we have $-1 \le x \le 1$ with $-\sqrt{1 - x^2} \le y \le \sqrt{1 - x^2}$ which describes the unit circle in the xy-plane. Thus,

$$\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{x^2+y^2}^{2-x^2-y^2} \left(x^2 + y^2\right)^{3/2} dz \, dy \, dx = \int_0^{2\pi} \int_0^1 \int_{r^2}^{2-r^2} \left(r^2\right)^{3/2} r \, dz \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 \left[r^4 z\right]_{z=r^2}^{z=2-r^2} dr \, d\theta = \int_0^{2\pi} \int_0^1 \left(2r^4 - r^6 - r^6\right) dr \, d\theta = \int_0^{2\pi} \left(\tfrac{2}{5} - \tfrac{2}{7}\right) d\theta = \tfrac{8\pi}{35}$$