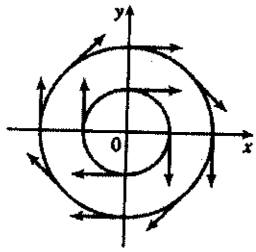
6. $\mathbf{F}(x,y) = \dfrac{y\,\mathbf{i} - x\,\mathbf{j}}{\sqrt{x^2+y^2}}$

All the vectors $\mathbf{F}(x,y)$ are unit vectors tangent to circles centered at the origin with radius $\sqrt{x^2+y^2}$.



11. $\mathbf{F}(x,y) = \langle y, x\rangle$ corresponds to graph III, since in the first quadrant all the vectors have positive x- and y-components, in the second quadrant all vectors have positive x-components and negative y-components, in the third quadrant all vectors have negative x- and y-components, and in the fourth quadrant all vectors have negative x-components and positive y-components.

12. $\mathbf{F}(x,y) = \langle 2x - 3y, 2x + 3y\rangle$ corresponds to graph IV, since as we move to the right (so x increases and y is constant), both the x- and the y-components of the vectors get larger, and as we move upward (so y increases and x is constant), the x-components decrease, while the y-components increase.

13. $\mathbf{F}(x,y) = \langle \sin x, \sin y\rangle$ corresponds to graph II, since the vector field is the same on each square of the form $[2n\pi, 2(n+1)\pi] \times [2m\pi, 2(m+1)\pi]$, m, n any integers.

14. $\mathbf{F}(x,y) = \left\langle \ln\left(1 + x^2 + y^2\right), x\right\rangle$ corresponds to graph I, since $\ln\left(1 + x^2 + y^2\right)$ is always positive, so all vectors point to the right.

15. $\mathbf{F}(x,y,z) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph IV, since all vectors have identical length and direction.

16. $\mathbf{F}(x,y,z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph I, since the horizontal vector components remain constant, but the vectors above the xy-plane point generally upward while the vectors below the xy-plane point generally downward.

17. $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\,\mathbf{k}$ corresponds to graph III; the projection of each vector onto the xy-plane is $x\,\mathbf{i} + y\,\mathbf{j}$, which points away from the origin, and the vectors point generally upward because their z-components are all 3.

18. $\mathbf{F}(x,y,z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ corresponds to graph II; each vector $\mathbf{F}(x,y,z)$ has the same length and direction as the position vector of the point (x,y,z), and therefore the vectors all point directly away from the origin.

29. $f(x,y) = xy \;\Rightarrow\; \nabla f(x,y) = y\,\mathbf{i} + x\,\mathbf{j}$. In the first quadrant, both components of each vector are positive, while in the third quadrant both components are negative. However, in the second quadrant each vector's x-component is positive while its y-component is negative (and vice versa in the fourth quadrant). Thus, ∇f is graph IV.

30. $f(x,y) = x^2 - y^2 \;\Rightarrow\; \nabla f(x,y) = 2x\,\mathbf{i} - 2y\,\mathbf{j}$. In the first quadrant, the x-component of each vector is positive while the y-component is negative. The other three quadrants are similar, where the x-component of each vector has the same sign as the x-value of its initial point, and the y-component has sign opposite that of the y-value of the initial point. Thus, ∇f is graph III.

31. $f(x,y) = x^2 + y^2 \;\Rightarrow\; \nabla f(x,y) = 2x\,\mathbf{i} + 2y\,\mathbf{j}$. Thus, each vector $\nabla f(x,y)$ has the same direction and twice the length of the position vector of the point (x,y), so the vectors all point directly away from the origin and their lengths increase as we move away from the origin. Hence, ∇f is graph II.

32. $f(x,y) = \sqrt{x^2+y^2} \;\Rightarrow\; \nabla f(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2+y^2}}\,\mathbf{j}$. Then

$|\nabla f(x,y)| = \dfrac{1}{\sqrt{x^2+y^2}}\sqrt{x^2+y^2} = 1$, so all vectors are unit vectors. In addition, each vector $\nabla f(x,y)$ has the same direction as the position vector of the point (x,y), so the vectors all point directly away from the origin. Hence, ∇f is graph I.