13.2 Line Integrals

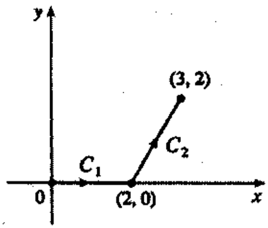
3. Parametric equatons for C are $x = 4\cos t$, $y = 4\sin t$, $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. Then

$$\int_C xy^4\,ds = \int_{-\pi/2}^{\pi/2} (4\cos t)(4\sin t)^4 \sqrt{(-4\sin t)^2 + (4\cos t)^2}\,dt$$

$$= \int_{-\pi/2}^{\pi/2} 4^5 \cos t \sin^4 t \sqrt{16\left(\sin^2 t + \cos^2 t\right)}\,dt$$

$$= 4^5 \int_{-\pi/2}^{\pi/2} \left(\sin^4 t \cos t\right)(4)\,dt = (4)^6 \left[\tfrac{1}{5}\sin^5 t\right]_{-\pi/2}^{\pi/2} = \frac{2 \cdot 4^6}{5} = 1638.4$$

5.



$C = C_1 + C_2$

On C_1: $x = x$, $y = 0 \;\Rightarrow\; dy = 0\,dx$, $0 \le x \le 2$.

On C_2: $x = x$, $y = 2x - 4 \;\Rightarrow\; dy = 2\,dx$, $2 \le x \le 3$.

Then

$$\int_C xy\,dx + (x - y)\,dy = \int_{C_1} xy\,dx + (x - y)\,dy + \int_{C_2} xy\,dx + (x - y)\,dy$$

$$= \int_0^2 (0 + 0)\,dx + \int_2^3 \left[\left(2x^2 - 4x\right) + (-x + 4)(2)\right] dx$$

$$= \int_2^3 \left(2x^2 - 6x + 8\right)\,dx = \frac{17}{3}$$

10. $\int_C yz\,dy + xy\,dz = \int_0^1 (t)\left(t^2\right)dt + \int_0^1 \sqrt{t}\,(t)\,2t\,dt = \int_0^1 \left(t^3 + 2t^{5/2}\right)dt = \left[\frac{1}{4}t^4 + \frac{4}{7}t^{7/2}\right]_0^1 = \frac{23}{28}$

14. Vectors starting on C_1 point in roughly the same direction as C_1, so the tangential component $\mathbf{F} \cdot \mathbf{T}$ is positive. Then $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_1} \mathbf{F} \cdot \mathbf{T}\,ds$ is positive. On the other hand, no vectors starting on C_2 point in the same direction as C_2, while some vectors point in roughly the opposite direction, so we would expect $\int_{C_2} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot \mathbf{T}\,ds$ to be negative.

17. $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \left\langle \sin t^3, \cos\left(-t^2\right), t^4 \right\rangle \cdot \left\langle 3t^2, -2t, 1 \right\rangle dt$

$$= \int_0^1 \left(3t^2 \sin t^3 - 2t\cos t^2 + t^4\right)dt = \left[-\cos t^3 - \sin t^2 + \tfrac{1}{5}t^5\right]_0^1 = \frac{6}{5} - \cos 1 - \sin 1$$

35. Let $\mathbf{F} = 185\,\mathbf{k}$. To parametrize the staircase, let

$x = 20\cos t$, $y = 20\sin t$, $z = \frac{90}{6\pi}t = \frac{15}{\pi}t$, $0 \le t \le 6\pi \;\Rightarrow$

$$W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^{6\pi} \langle 0, 0, 185\rangle \cdot \left\langle -20\sin t, 20\cos t, \tfrac{15}{\pi}\right\rangle dt = (185)\tfrac{15}{\pi}\int_0^{6\pi} dt = (185)(90)$$

$\approx 1.67 \times 10^4$ ft-lb