6. (a) $\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \\ x^2+y^2+z^2 & x^2+y^2+z^2 & x^2+y^2+z^2 \end{vmatrix}$

$= \dfrac{1}{(x^2+y^2+z^2)^2}\left[(-2yz+2yz)\,\mathbf{i} - (-2xz+2xz)\,\mathbf{j} + (-2xy+2xy)\,\mathbf{k}\right] = 0$

(b) $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F} = \dfrac{\partial}{\partial x}\left(\dfrac{x}{x^2+y^2+z^2}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{y}{x^2+y^2+z^2}\right) + \dfrac{\partial}{\partial z}\left(\dfrac{z}{x^2+y^2+z^2}\right)$

$= \dfrac{x^2+y^2+z^2-2x^2}{(x^2+y^2+z^2)^2} + \dfrac{x^2+y^2+z^2-2y^2}{(x^2+y^2+z^2)^2} + \dfrac{x^2+y^2+z^2-2z^2}{(x^2+y^2+z^2)^2}$

$= \dfrac{x^2+y^2+z^2}{(x^2+y^2+z^2)^2} = \dfrac{1}{x^2+y^2+z^2}$

8. If the vector field is $\mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$, then we know $R = 0$. In addition, P and Q don't vary in the z-direction, so $\dfrac{\partial R}{\partial x} = \dfrac{\partial R}{\partial y} = \dfrac{\partial R}{\partial z} = \dfrac{\partial P}{\partial z} = \dfrac{\partial Q}{\partial z} = 0$. As x increases, the x-component of each vector of $\mathbf{F}$ increases while the y-component remains constant, so $\dfrac{\partial P}{\partial x} > 0$ and $\dfrac{\partial Q}{\partial x} = 0$. Similarly, as y increases, the y-component of each vector increases while the x-component remains constant, so $\dfrac{\partial Q}{\partial y} > 0$ and $\dfrac{\partial P}{\partial y} = 0$.

(a) $\text{div } \mathbf{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + 0 > 0$

(b) $\text{curl } \mathbf{F} = \left(\dfrac{\partial R}{\partial y} - \dfrac{\partial Q}{\partial z}\right)\mathbf{i} + \left(\dfrac{\partial P}{\partial z} - \dfrac{\partial R}{\partial x}\right)\mathbf{j} + \left(\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y}\right)\mathbf{k}$

$= (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = 0$

10. (a) $\text{curl } f = \nabla \times f$ is meaningless because f is a scalar field.

(b) $\text{grad } f$ is a vector field.

(c) $\text{div } \mathbf{F}$ is a scalar field.

(d) $\text{curl }(\text{grad } f)$ is a vector field.

(e) $\text{grad } \mathbf{F}$ is meaningless because $\mathbf{F}$ is not a scalar field.

(f) $\text{grad }(\text{div } \mathbf{F})$ is a vector field.

(g) $\text{div}(\text{grad } f)$ is a scalar field.

(h) $\text{grad }(\text{div } f)$ is meaningless because f is a scalar field.

(i) $\text{curl }(\text{curl } \mathbf{F})$ is a vector field.

(j) $\text{div}(\text{div } \mathbf{F})$ is meaningless because $\text{div } \mathbf{F}$ is a scalar field.

(k) $(\text{grad } f) \times (\text{div } \mathbf{F})$ is meaningless because $\text{div } \mathbf{F}$ is a scalar field.

(l) $\text{div}(\text{curl }(\text{grad } f))$ is a scalar field.

13. $\text{curl } \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & x^2+2yz & y^2 \end{vmatrix} = (2y-2y)\,\mathbf{i} - (0-0)\,\mathbf{j} + (2x-2x)\,\mathbf{k} = 0$, $\mathbf{F}$ is defined on all

of $\mathbb{R}^3$, and the partial derivatives of the component functions are continuous, so $\mathbf{F}$ is conservative. Thus there exists a function f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = 2xy$ implies $f(x,y,z) = x^2y + g(y,z)$ and $f_y(x,y,z) = x^2 + g_y(y,z)$. But $f_y(x,y,z) = x^2 + 2yz$, so $g(y,z) = y^2z + h(z)$ and $f(x,y,z) = x^2y + y^2z + h(z)$. Thus $f_z(x,y,z) = y^2 + h'(z)$ but $f_z(x,y,z) = y^2$ so $h(z) = K$ and $f(x,y,z) = x^2y + y^2z + K$.

19. $\text{curl } \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f(x) & g(y) & h(z) \end{vmatrix} = (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = 0$.

Hence $\mathbf{F} = f(x)\,\mathbf{i} + g(y)\,\mathbf{j} + h(z)\,\mathbf{k}$ is irrotational.