4. On the surface, $f(x,y,z) = g\left(\sqrt{x^2+y^2+z^2}\right) = g(2) = -5$. So since the area of a sphere is $4\pi r^2$, $\iint_S f(x,y,z)\,dS = \iint_S g(2)\,dS = -5\iint_S dS = -5\left[4\pi(2)^2\right] = -80\pi$.

5. $\mathbf{r}(u,v) = uv\,\mathbf{i} + (u+v)\,\mathbf{j} + (u-v)\,\mathbf{k}$, $u^2+v^2 \le 1$ and $|\mathbf{r}_u \times \mathbf{r}_v| = \sqrt{4+2u^2+2v^2}$ (see Exercise 12.6.7). Then

$$\iint_S yx\,dS = \iint_{u^2+v^2\le 1} (u^2-v^2)\sqrt{4+2u^2+2v^2}\,dA = \int_0^{2\pi}\int_0^1 r^2(\cos^2\theta - \sin^2\theta)\sqrt{4+2r^2}\,r\,dr\,d\theta$$
$$= \left[\int_0^{2\pi}(\cos^2\theta - \sin^2\theta)\,d\theta\right]\left[\int_0^1 r^3\sqrt{4+2r^2}\,dr\right] = 0$$

since the first integral is 0.

14. Here S consists of three surfaces: S_1, the lateral surface of the cylinder, S_2, the front formed by the plane $x+y=2$; and the back, S_3, in the plane $y=0$. On S_1: using cylindrical coordinates, $\mathbf{r}(\theta,y) = \sin\theta\,\mathbf{i} + y\,\mathbf{j} + \cos\theta\,\mathbf{k}$, $0 \le \theta \le 2\pi$, $0 \le y \le 2-\sin\theta$, $|\mathbf{r}_\theta \times \mathbf{r}_y| = 1$ and $\iint_{S_1} xy\,dS = \int_0^{2\pi}\int_0^{2-\sin\theta} (\sin\theta)\,y\,dy\,d\theta = \int_0^{2\pi}\left[2\sin\theta - 2\sin^2\theta + \frac{1}{2}\sin^3\theta\right]d\theta = -2\pi$.

On S_2: $\mathbf{r}(x,z) = x\,\mathbf{i} + (2-x)\,\mathbf{j} + z\,\mathbf{k}$ and $|\mathbf{r}_x \times \mathbf{r}_z| = |-\mathbf{i}-\mathbf{j}| = \sqrt{2}$, where $x^2+z^2 \le 1$ and

$$\iint_{S_2} xy\,dS = \iint_{x^2+z^2\le 1} x(2-x)\sqrt{2}\,dA = \int_0^{2\pi}\int_0^1 \sqrt{2}\,(2r\sin\theta - r^2\sin^2\theta)\,r\,dr\,d\theta$$
$$= \sqrt{2}\int_0^{2\pi}\left[\tfrac{2}{3}\sin\theta - \tfrac{1}{4}\sin^2\theta\right]d\theta = -\tfrac{\sqrt{2}}{4}\pi$$

On S_3: $y=0$ so $\iint_{S_3} xy\,dS = 0$. Hence $\iint_S xy\,dS = -2\pi - \frac{\sqrt{2}}{4}\pi = -\frac{1}{4}(8+\sqrt{2})\pi$.

19. $\mathbf{F}(x,y,z) = xy\,\mathbf{i} + yz\,\mathbf{j} + zx\,\mathbf{k}$, $z = g(x,y) = 4 - x^2 - y^2$, and D is the square $[0,1]\times[0,1]$, so by Equation 10

$$\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D \left[-xy(-2x) - yz(-2y) + zx\right]dA$$
$$= \int_0^1\int_0^1 \left[2x^2y + 2y^2(4-x^2-y^2) + x(4-x^2-y^2)\right]dy\,dx$$
$$= \int_0^1 \left(\tfrac{1}{3}x^2 + \tfrac{11}{3}x - x^3 + \tfrac{34}{15}\right)dx = \tfrac{713}{180}$$

25. Let S_1 be the paraboloid $y = x^2+z^2$, $0 \le y \le 1$ and S_2 the disk $x^2+z^2 \le 1$, $y=1$. Since S is a closed surface, we use the outward orientation. On S_1: $\mathbf{F}(\mathbf{r}(x,z)) = (x^2+z^2)\,\mathbf{j} - z\,\mathbf{k}$ and $\mathbf{r}_x \times \mathbf{r}_z = 2x\,\mathbf{i} - \mathbf{j} + 2z\,\mathbf{k}$

(since the $\mathbf{j}$-component must be negative on S_1). Then

$$\iint_{S_1} \mathbf{F}\cdot d\mathbf{S} = \iint_{x^2+z^2\le 1}\left[-(x^2+z^2) - 2z^2\right]dA = -\int_0^{2\pi}\int_0^1 (r^2 + 2r^2\cos^2\theta)\,r\,dr\,d\theta$$
$$= -\int_0^{2\pi}\tfrac{1}{4}(1+2\cos^2\theta)\,d\theta = -\left(\tfrac{\pi}{2}+\tfrac{\pi}{2}\right) = -\pi$$

On S_2: $\mathbf{F}(\mathbf{r}(x,z)) = \mathbf{j} - z\,\mathbf{k}$ and $\mathbf{r}_z \times \mathbf{r}_x = \mathbf{j}$. Then $\iint_{S_2} \mathbf{F}\cdot d\mathbf{S} = \iint_{x^2+z^2\le 1}(1)\,dA = \pi$. Hence $\iint_S \mathbf{F}\cdot d\mathbf{S} = -\pi + \pi = 0$.

34. S is given by $\mathbf{r}(x,y) = x\,\mathbf{i} + y\,\mathbf{j} + \sqrt{x^2+y^2}\,\mathbf{k}$, $|\mathbf{r}_x \times \mathbf{r}_y| = \sqrt{1 + \dfrac{x^2+y^2}{x^2+y^2}} = \sqrt{2}$ so

$$m = \iint_S \left(10 - \sqrt{x^2+y^2}\right)dS = \iint_{1\le x^2+y^2\le 16}\left(10 - \sqrt{x^2+y^2}\right)\sqrt{2}\,dA$$
$$= \int_0^{2\pi}\int_1^4 \sqrt{2}\,(10-r)\,r\,dr\,d\theta = 2\pi\sqrt{2}\left[5r^2 - \tfrac{1}{3}r^3\right]_1^4 = 108\sqrt{2}\pi$$