3. The boundary curve C is the circle $x^2 + y^2 = 4$, $z = 0$ oriented in the counterclockwise direction. The vector equation is $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$, $0 \le t \le 2\pi$, so $\mathbf{r}'(t) = -2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$ and $\mathbf{F}(\mathbf{r}(t)) = (2\cos t)^2 e^{(2\sin t)(0)}\,\mathbf{i} + (2\sin t)^2 e^{(2\cos t)(0)}\,\mathbf{j} + (0)^2 e^{(2\cos t)(2\sin t)}\,\mathbf{k} = 4\cos^2 t\,\mathbf{i} + 4\sin^2 t\,\mathbf{j}$. Then, by Stokes' Theorem,

$$\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_0^{2\pi} \mathbf{F}(\mathbf{r}(t))\cdot\mathbf{r}'(t)\,dt = \int_0^{2\pi}\left(-8\cos^2 t\sin t + 8\sin^2 t\cos t\right)dt$$
$$= 8\left[\tfrac{1}{3}\cos^3 t + \tfrac{1}{3}\sin^3 t\right]_0^{2\pi} = 0$$

5. C is the square in the plane $z = -1$. By (3), $\iint_{S_1} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_{S_2} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$ where S_1 is the original cube without the bottom and S_2 is the bottom face of the cube. $\operatorname{curl}\mathbf{F} = x^2 z\,\mathbf{i} + (xy - 2xyz)\,\mathbf{j} + (y - xz)\,\mathbf{k}$. For S_2, we choose $\mathbf{n} = \mathbf{k}$ so that C has the same orientation for both surfaces. Then $\operatorname{curl}\mathbf{F}\cdot\mathbf{n} = y - xz = x + y$ on S_2, where $z = -1$. Thus $\iint_{S_2} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \int_{-1}^{1}\int_{-1}^{1}(x + y)\,dx\,dy = 0$ so $\iint_{S_1} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = 0$.

7. $\operatorname{curl}\mathbf{F} = -2z\,\mathbf{i} - 2x\,\mathbf{j} - 2y\,\mathbf{k}$ and we take the surface S to be the planar region enclosed by C, so S is the portion of the plane $x + y + z = 1$ over $D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le 1 - x\}$. Since C is oriented counterclockwise, we orient S upward. Using Equation 13.6.10, we have $z = g(x,y) = 1 - x - y$, $P = -2z$, $Q = -2x$, $R = -2y$, and

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \iint_D \left[-(-2z)(-1) - (-2x)(-1) + (-2y)\right] dA$$
$$= \int_0^1\int_0^{1-x}(-2)\,dy\,dx = -2\int_0^1 (1 - x)\,dx = -1$$

14. The plane intersects the coordinate axes at $x = 1$, $y = z = 2$ so the boundary curve C consists of the three line segments C_1: $\mathbf{r}_1(t) = (1 - t)\,\mathbf{i} + 2t\,\mathbf{j}$, $0 \le t \le 1$, C_2: $\mathbf{r}_2(t) = (2 - 2t)\,\mathbf{j} + 2t\,\mathbf{k}$, $0 \le t \le 1$, C_3: $\mathbf{r}_3(t) = t\,\mathbf{i} + (2 - 2t)\,\mathbf{k}$, $0 \le t \le 1$. Then

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \int_0^1 [(1 - t)\,\mathbf{i} + 2t\,\mathbf{j}]\cdot(-\mathbf{i} + 2\mathbf{j})\,dt + \int_0^1 [(2 - 2t)\,\mathbf{j}]\cdot(-2\mathbf{j} + 2\mathbf{k})\,dt + \int_0^1 (t\,\mathbf{i})\cdot(\mathbf{i} - 2\mathbf{k})\,dt$$
$$= \int_0^1 (5t - 1)\,dt + \int_0^1 (4t - 4)\,dt + \int_0^1 t\,dt = \tfrac{3}{2} - 2 + \tfrac{1}{2} = 0$$

Now $\operatorname{curl}\mathbf{F} = xz\,\mathbf{i} - yz\,\mathbf{j}$, so by Equation 13.6.10 with $z = g(x,y) = 2 - 2x - y$ we have

$$\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \iint_D [-x(2 - 2x - y)(-2) + y(2 - 2x - y)(-1)]\,dA$$
$$= \int_0^1\int_0^{2-2x}\left(4x - 4x^2 - 2y + y^2\right)dy\,dx$$
$$= \int_0^1 \left[4x(2 - 2x) - 4x^2(2 - 2x) - (2 - 2x)^2 + \tfrac{1}{3}(2 - 2x)^3\right]dx$$
$$= \int_0^1 \left(\tfrac{16}{3}x^3 - 12x^2 + 8x - \tfrac{4}{3}\right)dx = \left[\tfrac{4}{3}x^4 - 4x^3 + 4x^2 - \tfrac{4}{3}x\right]_0^1 = 0$$

19. Assume S is centered at the origin with radius a and let H_1 and H_2 be the upper and lower hemispheres, respectively, of S. Then $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \iint_{H_1} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} + \iint_{H_2} \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = \oint_{C_1} \mathbf{F}\cdot d\mathbf{r} + \oint_{C_2} \mathbf{F}\cdot d\mathbf{r}$ by Stokes' Theorem. But C_1 is the circle $x^2 + y^2 = a^2$ oriented in the counterclockwise direction while C_2 is the same circle oriented in the clockwise direction. Hence $\oint_{C_2} \mathbf{F}\cdot d\mathbf{r} = -\oint_{C_1} \mathbf{F}\cdot d\mathbf{r}$ so $\iint_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S} = 0$ as desired.