

Assignment 4

§9.5 Equations of Lines + Planes

p.1

4. This line has the same direction as the given line, $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Here $\mathbf{r}_0 = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$, so a vector equation is $\mathbf{r} = (0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}) + t(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 2t\mathbf{i} - t\mathbf{j} + 3t\mathbf{k}$ and parametric equations are $x = 2t, y = -t, z = 3t$.
5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n} = (1, 3, 1)$. So $\mathbf{r}_0 = \mathbf{i} + 6\mathbf{k}$, and we can take $\mathbf{v} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. Then a vector equation is $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k}) = (1+t)\mathbf{i} + 3t\mathbf{j} + (6+t)\mathbf{k}$, and parametric equations are $x = 1+t, y = 3t, z = 6+t$.
20. $\mathbf{j} + 2\mathbf{k} = \langle 0, 1, 2 \rangle$ is a normal vector to the plane and $(4, 0, -3)$ is a point on the plane, so setting $a = 0, b = 1, c = 2, x_0 = 4, y_0 = 0, z_0 = -3$ in Equation 6 gives $0(x - 4) + 1(y - 0) + 2(z - (-3)) = 0$ or $y + 2z + 6 = 0$ to be an equation of the plane.
36. The plane will contain all perpendicular bisectors of the line segment joining the two points. Thus, a point in the plane is $P_0 = (-1, -1, 2)$, the midpoint of the line segment joining the two given points, and a normal to the plane is $\mathbf{n} = \langle 6, -6, 2 \rangle$, the vector connecting the two points. So an equation of the plane is $6(x + 1) - 6(y + 1) + 2(z - 2) = 0$ or $3x - 3y + z = 2$.
48. Put $y = z = 0$ in the equation of the first plane to get the point $(\frac{4}{3}, 0, 0)$ on the plane. Because the planes are parallel the distance D between them is the distance from $(\frac{4}{3}, 0, 0)$ to the second plane. By Equation 8,
- $$D = \frac{|1(\frac{4}{3}) + 2(0) - 3(0) - 1|}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{3\sqrt{14}}$$
49. The distance between two parallel planes is the same as the distance between a point on one of the planes and the other plane. Let $P_0 = (x_0, y_0, z_0)$ be a point on the plane given by $ax + by + cz + d_1 = 0$. Then $ax_0 + by_0 + cz_0 + d_1 = 0$ and the distance between P_0 and the plane given by $ax + by + cz + d_2 = 0$ is, from Equation 8, $D = \frac{|ax_0 + by_0 + cz_0 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.