Assignment 4

§9.5 Equations of Lines + Planes

4. This line has the same direction as the given line, $\mathbf{v} = 2\,\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}$. Here $\mathbf{r}_0 = 0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}$, so a vector equation is $\mathbf{r} = (0\,\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k}) + t\,(2\,\mathbf{i} - \mathbf{j} + 3\,\mathbf{k}) = 2t\,\mathbf{i} - t\,\mathbf{j} + 3t\,\mathbf{k}$ and parametric equations are $x = 2t, y = -t, z = 3t$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n} = \langle 1, 3, 1 \rangle$. So $\mathbf{r}_0 = \mathbf{i} + 6\,\mathbf{k}$, and we can take $\mathbf{v} = \mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$. Then a vector equation is $\mathbf{r} = (\mathbf{i} + 6\,\mathbf{k}) + t(\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) = (1 + t)\,\mathbf{i} + 3t\,\mathbf{j} + (6 + t)\,\mathbf{k}$, and parametric equations are $x = 1 + t, y = 3t, z = 6 + t$.

---

20. $\mathbf{j} + 2\,\mathbf{k} = \langle 0, 1, 2 \rangle$ is a normal vector to the plane and $(4, 0, -3)$ is a point on the plane, so setting $a = 0$, $b = 1$, $c = 2$, $x_0 = 4$, $y_0 = 0$, $z_0 = -3$ in Equation 6 gives $0(x - 4) + 1(y - 0) + 2\,[z - (-3)] = 0$ or $y + 2z = -6$ to be an equation of the plane.

36. The plane will contain all perpendicular bisectors of the line segment joining the two points. Thus, a point in the plane is $P_0 = (-1, -1, 2)$, the midpoint of the line segment joining the two given points, and a normal to the plane is $\mathbf{n} = \langle 6, -6, 2 \rangle$, the vector connecting the two points. So an equation of the plane is $6(x + 1) - 6(y + 1) + 2(z - 2) = 0$ or $3x - 3y + z = 2$.

48. Put $y = z = 0$ in the equation of the first plane to get the point $(\frac{4}{3}, 0, 0)$ on the plane. Because the planes are parallel the distance D between them is the distance from $(\frac{4}{3}, 0, 0)$ to the second plane. By Equation 8,

$$D = \frac{\left|1\left(\frac{4}{3}\right) + 2(0) - 3(0) - 1\right|}{\sqrt{1^2 + 2^2 + (-3)^2}} = \frac{1}{3\sqrt{14}}.$$

49. The distance between two parallel planes is the same as the distance between a point on one of the planes and the other plane. Let $P_0 = (x_0, y_0, z_0)$ be a point on the plane given by $ax + by + cz + d_1 = 0$. Then $ax_0 + by_0 + cz_0 + d_1 = 0$ and the distance between P_0 and the plane given by $ax + by + cz + d_2 = 0$ is, from Equation 8, $D = \dfrac{|ax_0 + by_0 + cz_0 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.