PROBLEM SET 1

(1) Consider the experiment of drawing one card at random from a standard deck of cards. The outcome space Ω is the collection of the 52 cards. Assume that one is equally likely to draw any of the cards. Let

$$\begin{aligned} A &= \{\, x : x \text{ is a jack, queen, or king}\}, \\ B &= \{\, x : x \text{ is a 9, 10, or jack and } x \text{ is red}\}, \\ C &= \{\, x : x \text{ is a club}\}, \\ D &= \{\, x : x \text{ is a diamond, a heart, or a spade}\}. \end{aligned}$$

Find
(a) $\mathbb{P}(A)$. (b) $\mathbb{P}(A \cap B)$. (c) $\mathbb{P}(A \cup B)$.
(d) $\mathbb{P}(C \cup D)$. (e) $\mathbb{P}(C \cap D)$.

(2) A fair coin is tossed four times, and a sequence of head and tails is observed.
(a) List each of the 16 sequences that compose the outcome space Ω.
(b) Let events A, B, C, and D be given by $A = \{\text{at least 3 heads}\}$, $B = \{\text{at most 2 heads}\}$. $C = \{\text{heads on the third toss}\}$, and $D = \{1 \text{ head and 3 tails}\}$. If the probability of each outcome in the outcome space is equally likely, find (i) $\mathbb{P}(A)$, (ii) $\mathbb{P}(A \cap B)$, (iii) $\mathbb{P}(B)$, (iv) $\mathbb{P}(A \cup C)$, (v) $\mathbb{P}(D)$, (vi) $\mathbb{P}(A \cup C)$, and (vii) $\mathbb{P}(B \cup D)$.

(3) With the aid of Venn diagrams show the following:
(a) $(A \cap B)^c = A^c \cup B^c$.
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Please use multiple colors when drawing the diagrams.

(4) Using Venn diagrams and the properties of the probability set function show that

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B). \tag{1}$$

Check that this holds for part (c) and (d) of problem 1.

(5) Anne is quite the markswoman. She owns two rifles and with one of them she is a crack shot and hits a target with probability 2/3 but with the other rifle she only hits a target with probabilty 1/2. On any given day she is equally likely to choose either rifle for target practice. On this particular day she chooses one of the rifles and shoots 600 shots. When the smoke clears you see that she has hit the target 350 times. Which rifle was more likely used? Explain.

Hint: Assume that she is using one of the rifles and calculate the probability of the event. The assumption that leads to the higher probability is a good guess. This general procedure is called a *maximum likelihood* estimate in statistics. Note that you might encounter numbers too large for your calculator, consider the relationship between these numbers and use the logarithm function to circumvent the limitations of your calculator.