PROBLEM SET 2

(1) A light bulb company has factories in two cities. The factory in city A produces $\frac{2}{3}$ of the company's light bulbs. The remainder are produced in city B, and of these, 1% are defective. Among all bulbs manufactured by the company, what proportion are not defective and made in city B?

(2) One AIDS test, common in the early nineties, is called ELISA. Among 1,000,000 people who are given the ELISA test, we can expect results similar to those given in the table.

	B_1:Carry AIDS Virus	B_2:Do Not Carry AIDS Virus	Totals
A_1:Test positive	4,885	73,630	78,515
A_2:Test negative	115	921,370	921,485
Totals	5,000	995,000	1,000,000

If one of these 1,000,000 people is selected at random, find the following probabilities:
(a) $\mathbb{P}(B_1)$ (b) $\mathbb{P}(A_1)$ (c) $\mathbb{P}(A_1|B_2)$ (d) $\mathbb{P}(B_1|A_1)$.

(3) Suppose two cards are dealt from a well shuffled deck of 52. What is the probability that the second card is a spade given that the first card is black?

(4) Three high schools have senior classes of size 100, 400, 500, respectively. Here are two schemes for selecting a student from among the three senior classes:

A: Make a list of all 1000 seniors, and choose a student from the list randomly where the probability of picking any student is the same.

B: Pick one of school randomly, where the probability of choosing any school is the same, and then pick a student from the senior class of that school at random.

Show that these two schemes are not probabilistically equivalent. Here is a third scheme:

C: Pick school i with probability p_i ($p_1 + p_2 + p_3 = 1$), then pick a student at random from the senior class in the school.

Find the probabilities p_1, p_2, and p_3 which make scheme C equivalent to scheme A.

(5) (a) If the events A and B are mutually exclusive, are A and B always independent? If the answer is no, can they ever be independent? Explain. Note that a diagram and one sentence will suffice.

(b) If $A \subset B$, can A and B ever be independent events? Explain. As above, a diagram and one sentence will suffice.