Problem Set ~~I~~ 2 Solutions

1. Proportion ~~defe~~ not defective AND made in B

$= \left(\text{Proportion made in } B\right) \times \left(\text{Proportion of those made in } B \text{ that are not defective}\right).$

$= \left(\frac{1}{3}\right)(.99)$

$= .33$ or 33%.

This is an example of $\mathbb{P}(A \cap B) = \mathbb{P}(B)\,\mathbb{P}(A|B)$ with probability having the relative frequency interpretation.

2. a) $\mathbb{P}(B_1) = \mathbb{P}(B_1 \cap A_1) + \mathbb{P}(B_1 \cap A_2)$ because A_1, A_2 partition the outcome space Ω.

$= \frac{4885 + 115}{1\,000\,000}$

$= \frac{5000}{1\,000\,000}$

$= .005.$

b) $\mathbb{P}(A_1) = \frac{78515}{1000\,000}$

$= .078515$

c) $\mathbb{P}(A_1|B_2) = \frac{\mathbb{P}(A_1 \cap B_2)}{\mathbb{P}(B_2)}$

$= \frac{73,630}{995,000}$

$\approx .074$

d. $\mathbb{P}(B_1 | A_1) = \dfrac{\mathbb{P}(B_1 \cap A_1)}{\mathbb{P}(A_1)}$

$= \dfrac{4885}{78{,}515}$

$\approx .0622$

3. Define the following events:

$A_1 = \{$ first card spade $\}$.

$A_2 = \{$ first card club $\}$.

$A = \{$ first card black $\}$.

$B = \{$ second card spade $\}$.

We're calculating $\mathbb{P}(B | A)$ in this problem.

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(B \cap A)}{\mathbb{P}(A)}$$

$$= \frac{\mathbb{P}(B \cap A_1) + \mathbb{P}(B \cap A_2)}{\mathbb{P}(A)} \quad \text{since } A_1 \cup A_2 = A \text{ and } A_1 \cap A_2 = \emptyset.$$

$$= \frac{\mathbb{P}(B|A_1)\,\mathbb{P}(A_1) + \mathbb{P}(B|A_2)\,\mathbb{P}(A_2)}{\mathbb{P}(A)}$$

$$= \frac{\left(\frac{12}{51}\right)\left(\frac{13}{52}\right) + \left(\frac{13}{51}\right)\left(\frac{13}{52}\right)}{\left(\frac{1}{2}\right)} = 2\left(\frac{25}{51}\right)\left(\frac{13}{52}\right)$$

$$\approx .245.$$

4. To show that these are not probabilistically equivalent, we show that the probability of choosing a student in the school of 100 students is different in the two schemes:

A: $\mathbb{P}(\text{student from 100 school}) = \frac{1}{1000}$

B: $\mathbb{P}(\text{student from 100 school}) =$

$= \mathbb{P}(\text{choosing 100 school})\ \mathbb{P}(\text{student from 100 school} \mid \text{100 school chosen})$

$= \left(\frac{1}{3}\right)\left(\frac{1}{100}\right)$

$= \frac{1}{300}$.

Thus, the two schemes are not probabilistically equiv.

If schools are chosen with prob p_i then students from those schools can be chosen with prob. $p_i\left(\frac{1}{n_i}\right)$ where n_i is the number of students in the i^{th} school. Thus, we need

$p_i / n_i = \frac{1}{1000}$ or in other words.

$p_1 = \frac{1}{10}$, $p_2 = \frac{4}{10}$, $p_3 = \frac{5}{10}$.

5 a) Events A and B are exclusive if $A \cap B = \emptyset$. In this case, $\mathbb{P}(A \cap B) = 0$. On the other hands independent events satisfy

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B).$$

so exclusive events are only independent if $\mathbb{P}(A) = 0$ or $\mathbb{P}(B) = 0$.

b) If $A \subset B$, then $A \cap B = A$. In this case,

$\mathbb{P}(A \cap B) = \mathbb{P}(A)$. Thus, if $A \subset B$, ~~are~~ then A, B are independent ~~(for A~~ only if $\mathbb{P}(B) = 1$.