PROBLEM SET 3

(1) The fraction of persons in a population who have a certain disease is .01. A diagnostic test is available to test for the disease. For a healthy person the chance of being falsely diagnosed as having the disease is .05, while for someone with the disease the chance of being falsely diagnosed as healthy is .2. Suppose the test is performed on a person selected at random from the population.

(a) What is the probability that the test shows a positive result (meaning the person is diagnosed as diseased, perhaps correctly or incorrectly)?

(b) What is the probability that a person selected at random is one who has the disease but is diagnosed healthy?

(c) What is the probability that the person is correctly diagnosed and is healthy?

(d) Suppose the test shows a positive result. What is the probability that the person tested actually has the disease?

(e) Do the above probabilities admit a long-run frequency interpretation? Explain.

(2) The following data are derived from the 1973 Final Natality Statistics Report issued by the National Center for Health Statistics. These data are pertinent to live births only.

Suppose that infants are classified as low birthweight if they have a birthweight $\leq$ 2500g and as a normal birthweight $\geq$ 2501g. Suppose that infants are also classified by length of gestation in the following four categories: $<$ 20 weeks, 20-27 weeks, 28-36 weeks, $>$ 36 weeks. Assume that the probabilities of the different periods of gestation are as given:

Length of Gestation	**Probability**
$<$ 20 weeks	.0004
20-27 weeks	.0059
28-36 weeks	.0855
$>$ 36 weeks	.9028

Also assume that the probability of being low birthweight given that the length of gestation is $<$ 20 weeks is .540, the probability of being low birthweight given that the length of gestation is 20-27 weeks is .813, the probability of being low birthweight given that the length of gestation is 28-36 weeks is .379, and the probability of being low birthweight given that the length of gestation is $>$ 36 weeks is .035.

(a) What is the probability of having a low birthweight infant?

(b) Show that the events {length of gestation $\leq$ 27 weeks} and {low birthweight} are not independent.

(c) What is the probability of having a length of gestation $\leq$ 36 weeks given that a child is low birthweight?

(3) An urn contains 4 white balls and 6 black balls. A ball is chosen at random, and its color is noted. The ball is then replaced, along with 3 more balls of the same color (so that there are now 13 balls in the urn). Then another ball is drawn from the urn.

(a) What is the probability that the second ball drawn is white?

(b) Given that the second ball is white, what is the probability that the first ball drawn is black?

$*$(c) Suppose the original contents of the urn are w white and b black balls, and that after a ball is drawn from the urn, it is replaced along with d more balls of the same color. In part (a), w was 4, b was 6, and d was 3. Show that the chance that the second ball drawn is white is $\frac{w}{w+b}$. (Note that the probability does not depend on the value of d.)

(4) Let X and Y be independent discrete random variables where X is uniformly distributed between 0 and N and Y is uniformly distributed between 0 and 1, i.e. $\mathbb{P}(X = k) = \frac{1}{N+1}$ and $\mathbb{P}(Y = 0) = \mathbb{P}(Y = 1) = \frac{1}{2}$. N is a natural number. Find the distribution functions for the following dicrete random variables:
(a) $Z = X^2$. (b) $W = X + Y$. (c) Now find the expectation $\mathbb{E}Z$ and $\mathbb{E}W$.

(5) Your calculus TF offers you the opportunity to play the following game if you pay him. The TF has an unfair coin that has a 1/3 chance of coming up heads and a 2/3 chance of coming up tails. Your TF will flip his coin three times and pay you one dollar for each time the coin comes up heads. How much would you be willing to pay your TF to play this game? Explain.