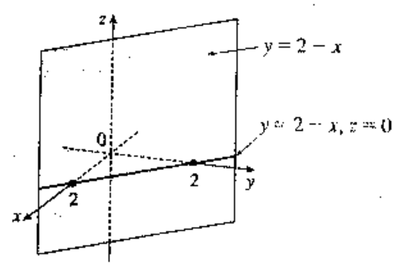
Assignment 1

§9.1 Three-Dimensional Coordinate Systems

5. The equation $x + y = 2$ represents the set of all points in $\mathbb{R}^3$ whose x- and y-coordinates have a sum of 2, or equivalently where $y = 2 - x$. This is the set $\{(x, 2 - x, z) \mid x \in \mathbb{R}, z \in \mathbb{R}\}$ which is a vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$.



8. (a) The distance from a point to the xy-plane is the absolute value of the z-coordinate of the point. Thus, the distance is $|-5| = 5$.

(b) Similarly, the distance is the absolute value of the x-coordinate of the point: $|3| = 3$.

(c) The distance is the absolute value of the y-coordinate of the point: $|7| = 7$.

(d) The point on the x-axis closest to $(3, 7, -5)$ is the point $(3, 0, 0)$. (Approach the x-axis perpendicularly.) The distance from $(3, 7, -5)$ to the x-axis is the distance between these two points:
$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60$.

(e) The point on the y-axis closest to $(3, 7, -5)$ is $(0, 7, 0)$. The distance between these points is
$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83$.

(f) The point on the z-axis closest to $(3, 7, -5)$ is $(0, 0, -5)$. The distance between these points is
$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62$.

10. An equation of the sphere with center $(6, 5, -2)$ and radius $\sqrt{7}$ is $(x-6)^2 + (y-5)^2 + [z-(-2)]^2 = (\sqrt{7})^2$ or $(x-6)^2 + (y-5) + (z+2)^2 = 7$. The intersection of this sphere with the xy-plane is the set of points on the sphere whose z-coordinate is 0. Putting $z = 0$ into the equation, we have $(x-6)^2 + (y-5)^2 = 3, z = 0$ which represents a circle in the xy-plane with center $(6, 5, 0)$ and radius $\sqrt{3}$. To find the intersection with the xz-plane, we set $y = 0$: $(x-6)^2 + (z+2)^2 = -18$. Since no points satisfy this equation, the sphere does not intersect the xz-plane. (Also note that the distance from the center of the sphere to the xz-plane is greater than the radius of the sphere.) Similarly, the sphere does not intersect the yz-plane since substituting $x = 0$ into the equation gives $(y-5)^2 + (z+2)^2 = -29$.

14. Completing squares in the equation gives $4(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 4z^2 = 1 + 4 + 16 \Rightarrow$ $4(x-1)^2 + 4(y+2)^2 + 4z^2 = 21 \Rightarrow (x-1)^2 + (y+2)^2 + z^2 = \frac{21}{4}$, which we recognize as an equation of a sphere with center $(1, -2, 0)$ and radius $\sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$.

28. The equation $xyz = 0$ is satisfied when any of x, y, or z is 0. Thus, the equation represents the region consisting of all points on the three coordinate planes $x = 0$, $y = 0$, and $z = 0$.

Assignment 1

31. This describes a region all of whose points have a distance to the origin which is greater than r, but smaller than R. So inequalities describing the region are $r < \sqrt{x^2 + y^2 + z^2} < R$, or $r^2 < x^2 + y^2 + z^2 < R^2$.

33. (a) To find the x- and y-coordinates of the point P, we project it onto L_2 and project the resulting point Q onto the x- and y-axes. To find the z-coordinate, we project P onto either the xz-plane or the yz-plane (using our knowledge of its x- or y-coordinate) and then project the resulting point onto the z-axis. (Or, we could draw a line parallel to QO from P to the z-axis.) The coordinates of P are $(2, 1, 4)$.

(b) A is the intersection of L_1 and L_2, B is directly below the y-intercept of L_2, and C is directly above the x-intercept of L_2.