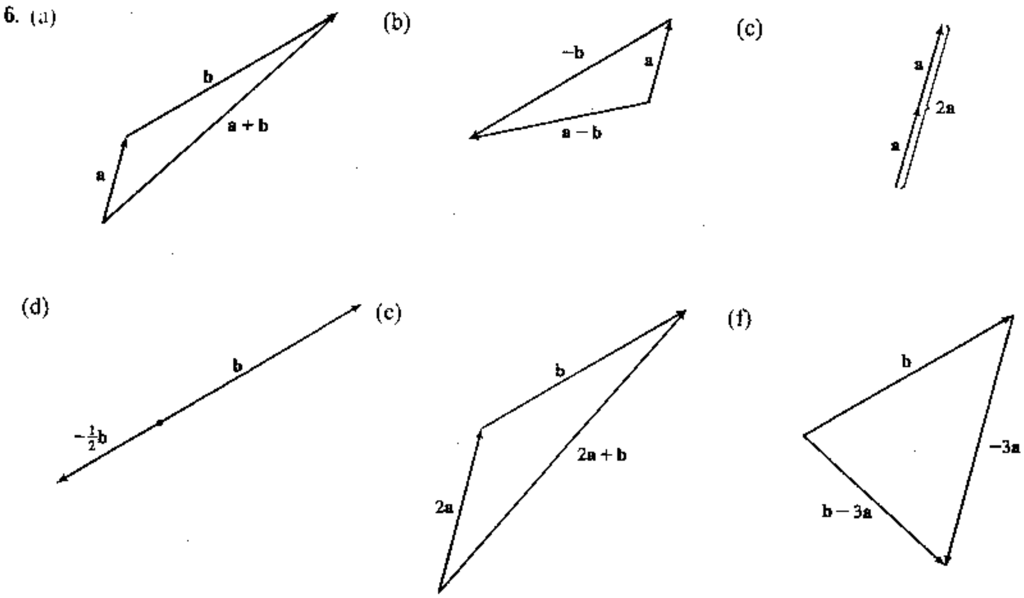
Assignment 2
§9.2 Vectors

6.



18. $|\mathbf{a}| = \sqrt{3^2 + 0^2 + (-2)^2} = \sqrt{13}$

$$\mathbf{a} + \mathbf{b} = (3\,\mathbf{i} - 2\,\mathbf{k}) + (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 4\,\mathbf{i} - \mathbf{j} - \mathbf{k}$$
$$\mathbf{a} - \mathbf{b} = (3\,\mathbf{i} - 2\,\mathbf{k}) - (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 2\,\mathbf{i} + \mathbf{j} - 3\,\mathbf{k}$$
$$2\mathbf{a} = 2(3\,\mathbf{i} - 2\,\mathbf{k}) = 6\,\mathbf{i} - 4\,\mathbf{k}$$
$$\begin{aligned} 3\mathbf{a} + 4\mathbf{b} &= 3(3\,\mathbf{i} - 2\,\mathbf{k}) + 4(\mathbf{i} - \mathbf{j} + \mathbf{k}) \\ &= 9\,\mathbf{i} - 6\,\mathbf{k} + 4\,\mathbf{i} - 4\,\mathbf{j} + 4\,\mathbf{k} \\ &= 13\,\mathbf{i} - 4\,\mathbf{j} - 2\,\mathbf{k} \end{aligned}$$

20. $|\langle -2, 4, 2\rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$, so a unit vector in the direction of $\langle -2, 4, 2\rangle$ is $\mathbf{u} = \frac{1}{2\sqrt{6}}\langle -2, 4, 2\rangle$. A vector in the same direction but with length 6 is
$$6\mathbf{u} = 6 \cdot \frac{1}{2\sqrt{6}}\langle -2, 4, 2\rangle = \left\langle -\frac{6}{\sqrt{6}}, \frac{12}{\sqrt{6}}, \frac{6}{\sqrt{6}} \right\rangle \text{ or } \langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\rangle.$$

24. Set up the coordinate axes so that north is the positive y-direction, and east is the positive x-direction. The wind is blowing at 50 km/h from the direction N 45° W, so that its velocity vector is 50 km/h S 45° E, which can be written as $\mathbf{v}_{\text{wind}} = 50(\cos 45°\,\mathbf{i} - \sin 45°\,\mathbf{j})$. With respect to the still air, the velocity vector of the plane is 250 km/h N 60° E, or equivalently $\mathbf{v}_{\text{plane}} = 250(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$. The velocity of the plane relative to the ground is

$$\begin{aligned} \mathbf{v} = \mathbf{v}_{\text{wind}} + \mathbf{v}_{\text{plane}} &= (50\cos 45° + 250\cos 30°)\,\mathbf{i} + (-50\sin 45° + 250\sin 30°)\,\mathbf{j} \\ &= \left(25\sqrt{2} + 125\sqrt{3}\right)\mathbf{i} + \left(125 - 25\sqrt{2}\right)\mathbf{j} \approx 251.9\,\mathbf{i} + 89.6\,\mathbf{j} \end{aligned}$$

The ground speed is $|\mathbf{v}| \approx \sqrt{(251.9)^2 + (89.6)^2} \approx 267$ km/h. The angle the velocity vector makes with the x-axis is $\theta \approx \tan^{-1}\frac{89.6}{251.9} \approx 20°$. Therefore, the true course of the plane is about N $(90 - 20)°$ E $=$ N 70° E.

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34. Let P_1 and P_2 be the points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively. Then $|\mathbf{r}-\mathbf{r}_1|+|\mathbf{r}-\mathbf{r}_2|$ is the sum of the distances from (x, y) to P_1 and P_2. Since this sum is constant, the set of points (x, y) represents an ellipse with foci P_1 and P_2. The condition $k > |\mathbf{r}_1 - \mathbf{r}_2|$ assures us that the ellipse is not degenerate.

§9.3 The Dot Product

8. $\mathbf{a}\cdot\mathbf{b} = (4\,\mathbf{j} - 3\,\mathbf{k})\cdot(2\,\mathbf{i} + 4\,\mathbf{j} + 6\,\mathbf{k}) = (0)(2) + (4)(4) + (-3)(6) = -2$

16. Let p, q and r be the angles at vertices P, Q and R. Then p is the angle between vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$, q is the angle between vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$, and r is the angle between vectors $\overrightarrow{RP}$ and $\overrightarrow{RQ}$. Thus

$$\cos p = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|} = \frac{\langle 2, 2, -9\rangle\cdot\langle 5, 5, -4\rangle}{\sqrt{89}\,\sqrt{66}} = \frac{56}{\sqrt{5874}}, \text{ so } p = \cos^{-1}\frac{56}{\sqrt{5874}} \approx 43^\circ;$$

$$\cos q = \frac{\overrightarrow{QP}\cdot\overrightarrow{QR}}{\left|\overrightarrow{QP}\right|\left|\overrightarrow{QR}\right|} = \frac{\langle -2, -2, 9\rangle\cdot\langle 3, 3, 5\rangle}{\sqrt{89}\,\sqrt{43}} = \frac{33}{\sqrt{3827}}, \text{ so } q = \cos^{-1}\frac{33}{\sqrt{3827}} \approx 58^\circ; \text{ and}$$

$r \approx 180^\circ - (43^\circ + 58^\circ) = 79^\circ$.

Alternate Solution: Apply the Law of Cosines three times as follows:

$$\cos p = \frac{\left|\overrightarrow{QR}\right|^2 - \left|\overrightarrow{PQ}\right|^2 - \left|\overrightarrow{PR}\right|^2}{2\left|\overrightarrow{PQ}\right|\left|\overrightarrow{PR}\right|}, \cos q = \frac{\left|\overrightarrow{PR}\right|^2 - \left|\overrightarrow{PQ}\right|^2 - \left|\overrightarrow{QR}\right|^2}{2\left|\overrightarrow{PQ}\right|\left|\overrightarrow{QR}\right|}, \cos r = \frac{\left|\overrightarrow{PQ}\right|^2 - \left|\overrightarrow{PR}\right|^2 - \left|\overrightarrow{QR}\right|^2}{2\left|\overrightarrow{PR}\right|\left|\overrightarrow{QR}\right|}.$$