18. $\langle -6, b, 2\rangle$ and $\langle b, b^2, b\rangle$ are orthogonal when $\langle -6, b, 2\rangle \cdot \langle b, b^2, b\rangle = 0 \quad \Leftrightarrow \quad (-6)(b) + (b)(b^2) + (2)(b) = 0$
$\Leftrightarrow \quad b^3 - 4b = 0 \quad \Leftrightarrow \quad b(b+2)(b-2) = 0 \quad \Leftrightarrow \quad b = 0$ or $b = \pm 2$.

25. $(\text{orth}_{\mathbf{a}}\,\mathbf{b}) \cdot \mathbf{a} = (\mathbf{b} - \text{proj}_{\mathbf{a}}\,\mathbf{b}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - (\text{proj}_{\mathbf{a}}\,\mathbf{b}) \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} - \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a} \cdot \mathbf{a}$

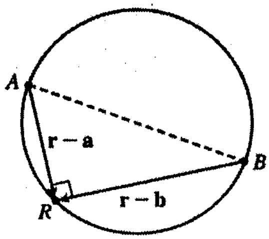
$$= \mathbf{b} \cdot \mathbf{a} - \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\,|\mathbf{a}|^2 = \mathbf{b} \cdot \mathbf{a} - \mathbf{a} \cdot \mathbf{b} = 0$$

So they are orthogonal by (2).

33. First note that $\mathbf{n} = \langle a, b\rangle$ is perpendicular to the line, because if $Q_1 = (a_1, b_1)$ and $Q_2 = (a_2, b_2)$ lie on the line, then $\mathbf{n} \cdot \overrightarrow{Q_1Q_2} = aa_2 - aa_1 + bb_2 - bb_1 = 0$, since $aa_2 + bb_2 = -c = aa_1 + bb_1$ from the equation of the line. Let $P_2 = (x_2, y_2)$ lie on the line. Then the distance from P_1 to the line is the absolute value of the scalar projection of $\overrightarrow{P_1P_2}$ onto $\mathbf{n}$. $\text{comp}_{\mathbf{n}}\left(\overrightarrow{P_1P_2}\right) = \dfrac{|\mathbf{n} \cdot \langle x_2 - x_1, y_2 - y_1\rangle|}{|\mathbf{n}|} = \dfrac{|ax_2 - ax_1 + by_2 - by_1|}{\sqrt{a^2 + b^2}} = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

since $ax_2 + by_2 = -c$. The required distance is $\dfrac{|3 \cdot -2 + -4 \cdot 3 + 5|}{\sqrt{3^2 + 4^2}} = \dfrac{13}{5}$.

34. $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$ implies that the vectors $\mathbf{r} - \mathbf{a}$ and $\mathbf{r} - \mathbf{b}$ are orthogonal. From the diagram (in which A, B and R are the terminal points of the vectors), we see that this implies that R lies on a sphere whose diameter is the line from A to B. The center of this circle is the midpoint of AB, that is,
$\frac{1}{2}(\mathbf{a} + \mathbf{b}) = \left\langle \frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2), \frac{1}{2}(a_3 + b_3)\right\rangle$, and its radius is
$\frac{1}{2}\,|\mathbf{a} - \mathbf{b}| = \frac{1}{2}\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$.
Or: Expand the given equation, substitute $\mathbf{r} \cdot \mathbf{r} = x^2 + y^2 + z^2$ and complete the squares.



37. Consider the H-C-H combination consisting of the sole carbon atom and the two hydrogen atoms that are at (1, 0, 0) and (0, 1, 0) (or any H-C-H combination, for that matter). Vector representations of the line segments emanating from the carbon atom and extending to these two hydrogen atoms are $\left\langle 1 - \frac{1}{2}, 0 - \frac{1}{2}, 0 - \frac{1}{2}\right\rangle = \left\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\rangle$ and $\left\langle 0 - \frac{1}{2}, 1 - \frac{1}{2}, 0 - \frac{1}{2}\right\rangle = \left\langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\rangle$. The bond angle, θ, is therefore given by

$$\cos\theta = \frac{\left\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\rangle \cdot \left\langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\rangle}{\left|\left\langle \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right\rangle\right| \left|\left\langle -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right\rangle\right|} = \frac{-\frac{1}{4} - \frac{1}{4} + \frac{1}{4}}{\sqrt{\frac{3}{4}}\sqrt{\frac{3}{4}}} = -\frac{1}{3} \quad \Rightarrow \quad \theta = \cos^{-1}\left(-\tfrac{1}{3}\right) \approx 109.5^\circ.$$

9.4 The Cross Product

8. $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -3 & 2 \\ 6 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -3 & 2 \\ 6 & 3 \end{vmatrix}\mathbf{k}$

$$= (2 - 6)\,\mathbf{i} - (-3 - 12)\,\mathbf{j} + (-9 - 12)\,\mathbf{k} = -4\,\mathbf{i} + 15\,\mathbf{j} - 21\,\mathbf{k}$$

Since $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \langle -4, 15, -21\rangle \cdot \langle -3, 2, 2\rangle = 12 + 30 - 42 = 0$, $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$.
Since $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \langle -4, 15, -21\rangle \cdot \langle 6, 3, 1\rangle = -24 + 45 - 21 = 0$, $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{b}$.

9. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ t & t^2 & t^3 \\ 1 & 2t & 3t^2 \end{vmatrix} = \begin{vmatrix} t^2 & t^3 \\ 2t & 3t^2 \end{vmatrix}\mathbf{i} - \begin{vmatrix} t & t^3 \\ 1 & 3t^2 \end{vmatrix}\mathbf{j} + \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix}\mathbf{k}$

$= (3t^4 - 2t^4)\,\mathbf{i} - (3t^3 - t^3)\,\mathbf{j} + (2t^2 - t^2)\,\mathbf{k} = t^4\,\mathbf{i} - 2t^3\,\mathbf{j} + t^2\,\mathbf{k}$

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = \langle t^4, -2t^3, t^2\rangle \cdot \langle t, t^2, t^3\rangle = t^5 - 2t^5 + t^5 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = \langle t^4, -2t^3, t^2\rangle \cdot \langle 1, 2t, 3t^2\rangle = t^4 - 4t^4 + 3t^4 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{b}$.

16. The parallelogram is determined by the vectors $\overrightarrow{KL} = \langle 0,1,3\rangle$ and $\overrightarrow{KN} = \langle 2,5,0\rangle$, so the area of parallelogram $KLMN$ is

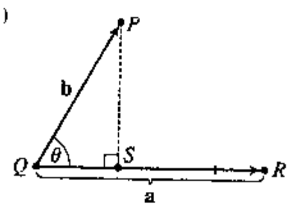
$\left|\overrightarrow{KL}\times\overrightarrow{KN}\right| = \left\| \begin{matrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 2 & 5 & 0 \end{matrix} \right\| = |(-15)\,\mathbf{i} - (-6)\mathbf{j} + (-2)\mathbf{k}| = |-15\,\mathbf{i} + 6\,\mathbf{j} - 2\,\mathbf{k}| = \sqrt{265} \approx 16.28.$

18. (a) $\overrightarrow{PQ} = \langle 1,1,3\rangle$ and $\overrightarrow{PR} = \langle 3,2,5\rangle$, so a vector orthogonal to the plane through P, Q, and R is $\overrightarrow{PQ}\times\overrightarrow{PR} = \langle (1)(5) - (3)(2), (3)(3) - (1)(5), (1)(2) - (1)(3)\rangle = \langle -1, 4, -1\rangle$ (or any scalar multiple thereof).

(b) The area of the parallelogram determined by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = |\langle -1,4,-1\rangle| = \sqrt{1+16+1} = \sqrt{18} = 3\sqrt{2}$, so the area of triangle PQR is $\frac{1}{2}\cdot 3\sqrt{2} = \frac{3}{2}\sqrt{2}$.

22. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 2 & 3 & -2 \\ 1 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = 2\begin{vmatrix} -1 & 0 \\ 0 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + (-2)\begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = -6 - 9 - 4 = -19$. So the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ is $|-19| = 19$ cubic units.

27. (a)



The distance between a point and a line is the length of the perpendicular from the point to the line, here $\left|\overrightarrow{PS}\right| = d$. But referring to triangle PQS, $d = \left|\overrightarrow{PS}\right| = \left|\overrightarrow{QP}\right|\sin\theta = |\mathbf{b}|\sin\theta$. But θ is the angle between $\overrightarrow{QP} = \mathbf{b}$ and $\overrightarrow{QR} = \mathbf{a}$. Thus by definition of the cross product, $\sin\theta = \dfrac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|}$ and so $d = |\mathbf{b}|\sin\theta = \dfrac{|\mathbf{b}|\,|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|\,|\mathbf{b}|} = \dfrac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|}$.

(b) $\mathbf{a} = \overrightarrow{QR} = \langle -1,-2,-1\rangle$ and $\mathbf{b} = \overrightarrow{QP} = \langle 1,-5,-7\rangle$. Then $\mathbf{a}\times\mathbf{b} = \langle (-2)(-7) - (-1)(-5), (-1)(1) - (-1)(-7), (-1)(-5) - (-2)(1)\rangle = \langle 9,-8,7\rangle$. Thus the distance is $d = \dfrac{|\mathbf{a}\times\mathbf{b}|}{|\mathbf{a}|} = \frac{1}{\sqrt{6}}\sqrt{81+64+49} = \sqrt{\frac{194}{6}} = \sqrt{\frac{97}{3}}$.