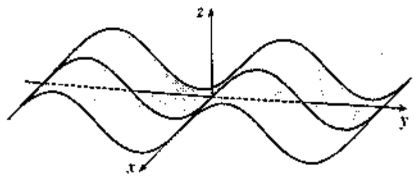
Assignment 5

§9.6 Functions & Surfaces

12. $z = \sin y$, a "wave".



15. All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x,y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x,y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

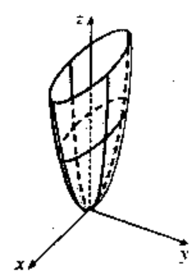
(c) $f(x,y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

(d) $f(x,y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \Rightarrow y = \pm x$, so it must be graph IV.

(e) $f(x,y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \Rightarrow y = x$, so it must be graph II.

(f) $f(x,y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

17. The equation of the graph is $z = x^2 + 9y^2$. The traces in $x = k$ are $z = 9y^2 + k^2$, a family of parabolas opening upward. In $y = k$, we have $z = x^2 + 9k^2$, again a family of parabolas opening upward. The traces in $z = k$ are $x^2 + 9y^2 = k$, a family of ellipses. The surface is an elliptic paraboloid.



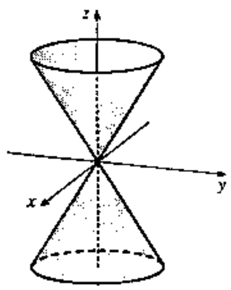
23. (a) In $\mathbb{R}^2$, $x^2+y^2=1$ represents a circle of radius 1 centered at the origin.

(b) In $\mathbb{R}^3$, the equation doesn't involve z, which means that any horizontal plane $z=k$ intersects the surface in a circle $x^2+y^2=1$, $z=k$. Thus the surface is a circular cylinder, made up of infinitely many shifted copies of the circle $x^2+y^2=1$, with axis the z-axis.

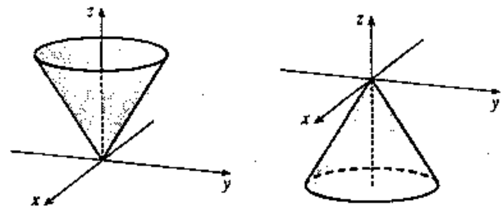
(c) In $\mathbb{R}^3$, $x^2+z^2=1$ also represents a circular cylinder of radius 1, this time with axis the y-axis.

24. (a) The traces of $z^2=x^2+y^2$ in $x=k$ are $z^2=y^2+k^2$, a family of hyperbolas, as are traces in $y=k$, $z^2=x^2+k^2$. Traces in $z=k$ are $x^2+y^2=k^2$, a family of circles.

(b) The surface is a circular cone with axis the z-axis.



(c) The graph of $f(x,y)=\sqrt{x^2+y^2}$ is the upper half of the cone in part (b), and the graph of $g(x,y)=-\sqrt{x^2+y^2}$ is the lower half.



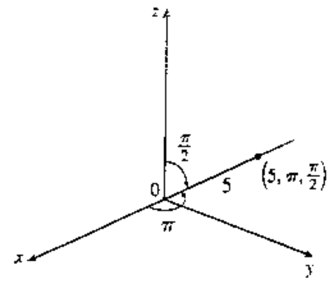
33. If (a,b,c) satisfies $z=y^2-x^2$, then $c=b^2-a^2$. L_1: $x=a+t$, $y=b+t$, $z=c+2(b-a)t$, L_2: $x=a+t$, $y=b-t$, $z=c-2(b+a)t$. Substitute the parametric equations of L_1 into the equation of the hyperbolic paraboloid in order to find the points of intersection: $z=y^2-x^2 \Rightarrow$ $c+2(b-a)t=(b+t)^2-(a+t)^2=b^2-a^2+2(b-a)t \Rightarrow c=b^2-a^2$. As this is true for all values of t, L_1 lies on $z=y^2-x^2$. Performing similar operations with L_2 gives: $z=y^2-x^2 \Rightarrow$ $c-2(b+a)t=(b-t)^2-(a+t)^2=b^2-a^2-2(b+a)t \Rightarrow c=b^2-a^2$. This tells us that all of L_2 also lies on $z=y^2-x^2$.

§9.7 Cylindrical & Spherical Coordinates

6. (a) $r^2=x^2+y^2=3^2+3^2=18$ so $r=\sqrt{18}=3\sqrt{2}$; $\tan\theta=\frac{y}{x}=\frac{3}{3}=1$ and the point $(3,3)$ is in the first quadrant of the xy-plane, so $\theta=\frac{\pi}{4}+2n\pi$; $z=-2$. Thus, one set of cylindrical coordinates is $(3\sqrt{2},\frac{\pi}{4},-2)$.

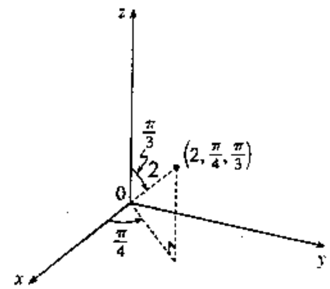
(b) $r^2=3^2+4^2=25$ so $r=5$; $\tan\theta=\frac{4}{3}$ and the point $(3,4)$ is in the first quadrant of the xy-plane, so $\theta=\tan^{-1}\frac{4}{3}+2n\pi\approx 0.93+2n\pi$; $z=5$. Thus, one set of cylindrical coordinates is $(5,\tan^{-1}\frac{4}{3},5)\approx(5,0.93,5)$.

8. (a)



$x = 5\sin\frac{\pi}{2}\cos\pi = -5$, $y = 5\sin\frac{\pi}{2}\sin\pi = 0$, $z = 5\cos\frac{\pi}{2} = 0$ so the point is $(-5, 0, 0)$ in rectangular coordinates.

(b)



$x = 2\sin\frac{\pi}{3}\cos\frac{\pi}{4} = \frac{\sqrt{6}}{2}$, $y = 2\sin\frac{\pi}{3}\sin\frac{\pi}{4} = \frac{\sqrt{6}}{2}$, $z = 2\cos\frac{\pi}{3} = 1$ so the point is $\left(\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}, 1\right)$ in rectangular coordinates.

16. Since $\rho\sin\phi = 2$ and $x = \rho\sin\phi\cos\theta$, $x = 2\cos\theta$. Also $y = \rho\sin\phi\sin\theta$ so $y = 2\sin\theta$. Then $x^2 + y^2 = 4\cos^2\theta + 4\sin^2\theta = 4$, a circular cylinder of radius 2 about the z-axis.

30. (a) The hollow ball is a spherical shell with outer radius 15 cm and inner radius 14.5 cm. If we center the ball at the origin of the coordinate system and use centimeters as the unit of measurement, then spherical coordinates conveniently describe the hollow ball as $14.5 \le \rho \le 15$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi$.

(b) If we position the ball as in part (a), one possibility is to take the half of the ball that is above the xy-plane which is described by $14.5 \le \rho \le 15$, $0 \le \theta \le 2\pi$, $0 \le \phi \le \pi/2$.

34. We begin by finding the positions of Los Angeles and Montréal in spherical coordinates, using the method described in the exercise:

Montréal	Los Angeles
$\rho = 3960$ mi	$\rho = 3960$ mi
$\theta = 360^\circ - 73.60^\circ = 286.40^\circ$	$\theta = 360^\circ - 118.25^\circ = 241.75^\circ$
$\phi = 90^\circ - 45.50^\circ = 44.50^\circ$	$\phi = 90^\circ - 34.06^\circ = 55.94^\circ$

Now we change the above to Cartesian coordinates using $x = \rho\cos\theta\sin\phi$, $y = \rho\sin\theta\sin\phi$ and $z = \rho\cos\phi$ to get two position vectors of length 3960 mi (since both cities must lie on the surface of the Earth). In particular:

Montréal: $\langle 783.67, -2662.67, 2824.47\rangle$ Los Angeles: $\langle -1552.80, -2889.91, 2217.84\rangle$

To find the angle α between these two vectors we use the dot product:

$\langle 783.67, -2662.67, 2824.47\rangle \cdot \langle -1552.80, -2889.91, 2217.84\rangle = (3960)$ [illegible] $\cos\alpha \Rightarrow \cos\alpha \approx 0.8126 \Rightarrow$ $\alpha \approx 0.6223$ rad. The great circle distance between the cities is $s = \rho\theta$ [illegible] 2464 mi.