MONDAY 1ST DECEMBER : THE FUNDAMENTAL THEOREM OF LINE INTEGRALS

Reading: sections 13.3 and 13.4
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Line Integrals

(1) Let $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$. Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where

(a) C is the upper half of the semicircle from $(1, 0)$ to $(-1, 0)$.
(b) C is the straight line segment from $(1, 0)$ to $(-1, 0)$.

(2) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the straight line segment from $(1, 0, -2)$ to $(4, 6, 3)$ and

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + (xy + 2z)\mathbf{k}$$

2. Conservative vector fields

(1) Are the following vector fields conservative? If so, find a function f such that $\mathbf{F} = \nabla f$.

(a)

$$\mathbf{F}(x, y) = xe^y\mathbf{i} + ye^x\mathbf{j}$$

(b)

$$\mathbf{F}(x, y) = (\sin(xy) + xy\cos(xy))\,\mathbf{i} + x^2\cos(xy)\mathbf{j}$$

3. A harder problem

Consider the vector field

$$\mathbf{F}(x, y) = \left(-\frac{y}{x^2 + y^2}\right)\mathbf{i} + \left(\frac{x}{x^2 + y^2}\right)\mathbf{j}$$

(1) Show that

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

(2) Show that $\mathbf{F}$ is not conservative.
(3) Why does this not violate Theorem 6 on page 940 of the textbook?