WEDNESDAY 10TH DECEMBER : STOKES' THEOREM

Reading: sections 13.6, 13.7
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Recap: Flux Integrals

(1) Let

$$\mathbf{F}(x, y, z) = y^3\mathbf{i} + \mathbf{j} + z\mathbf{k}$$

Let S be the part of the surface $z = x^2 + y^2$ below the plane $z = 1$, oriented upwards.
(a) Compute curl($\mathbf{F}$)
(b) Compute the flux of curl($\mathbf{F}$) through the surface S.

(2) Let

$$\mathbf{F}(x, y, z) = z\mathbf{i} + x\mathbf{j} + y\mathbf{k}$$

Compute the flux of curl($\mathbf{F}$) through the upper half of the sphere of radius 2 about the origin, oriented upwards.

2. Stokes' Theorem I : turning surface integrals into line integrals

(1) Use Stokes' Theorem to do the first problem in Section 1 above.

(2) Let

$$\mathbf{F}(x, y, z) = x^2 e^{yz}\mathbf{i} + y^2 e^{xz}\mathbf{j} + z^2 e^{xy}\mathbf{k}$$

Use Stokes' Theorem to find the flux of curl($\mathbf{F}$) through the upper half of the sphere of radius 2 about the origin, oriented upwards.

3. Stokes' Theorem II : turning line integrals into surface integrals

(1) Find the line integral of

$$\mathbf{F}(x, y, z) = \langle z, 2x, 3y \rangle$$

around the curve which is the intersection of the plane $z = 2 + x$ with the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above.

(2) Find the line integral of

$$\mathbf{F}(x, y, z) = \left\langle x^2 + y, y^2 + z, z^2 + x \right\rangle$$

counterclockwise (as viewed from above) around the triangle with vertices $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.