

$$\text{so } \int_C \underline{F} \cdot d\underline{r} = \iint_S (\text{curl } \underline{F}) \cdot d\underline{S} = \iint_S (\text{curl } \underline{F}) \cdot \underline{n} \, dS$$

But the normal vector field \underline{n} to the plane $z-x=2$ is $\frac{1}{\sqrt{2}} \langle -1, 0, 1 \rangle$, and so

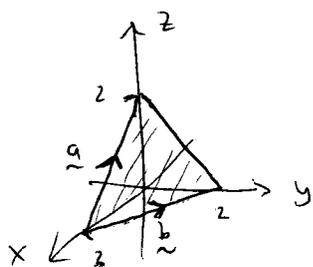
$$\begin{aligned} \int_C \underline{F} \cdot d\underline{r} &= \frac{1}{\sqrt{2}} \iint_S \langle 3, 1, 2 \rangle \cdot \langle -1, 0, 1 \rangle \, dS \\ &= -\frac{1}{\sqrt{2}} \iint_S 1 \, dS = -\frac{1}{\sqrt{2}} \text{area}(S') \end{aligned}$$

To calculate the area of S' , parametrize it by $x=u \quad y=v \quad z=2+u$ for $(u,v) \in D = \{(u,v) : u^2+v^2 \leq 1\}$.

$$\begin{aligned} \int_C \underline{F} \cdot d\underline{r} &= -\frac{1}{\sqrt{2}} \text{area}(S') \\ &= -\frac{1}{\sqrt{2}} \iint_D |\underline{r}_u \times \underline{r}_v| \, dA = \cancel{-\frac{1}{\sqrt{2}}} \iint_D 1 \, dA \\ &= \cancel{-\frac{1}{\sqrt{2}}} \text{area}(D) = \cancel{-\frac{1}{\sqrt{2}}} \pi = \underline{\underline{-\pi}} \end{aligned}$$

$\begin{cases} \underline{r}_u = \langle 1, 0, 1 \rangle \\ \underline{r}_v = \langle 0, 1, 0 \rangle \\ \underline{r}_u \times \underline{r}_v = \langle -1, 0, 1 \rangle \end{cases}$

(2) The triangle is the boundary of the part of the plane $x+y+z=2$ in the first octant; call this surface S' . Since C is oriented counter-clockwise, S' is oriented upwards. ~~the normal vector field to S' pointing in the right direction is~~ The normal vector field to S' pointing in the right direction is $\frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle$.



$$\text{curl } \underline{F} = \langle -1, -1, -1 \rangle, \text{ so}$$

$$\begin{aligned} \int_C \underline{F} \cdot d\underline{r} &= \iint_{S'} (\text{curl } \underline{F}) \cdot \underline{n} \, dS \\ &= \iint_{S'} -\sqrt{3} \, dS \\ &= -\sqrt{3} \text{area}(S) \\ &= -\frac{\sqrt{3}}{2} |\underline{a} \times \underline{b}| \quad (\text{see picture}) \\ &= -\frac{\sqrt{3}}{2} | \langle -2, 0, 2 \rangle \times \langle -2, 2, 0 \rangle | \\ &= \underline{\underline{-6}} \end{aligned}$$