

**FRIDAY 12TH DECEMBER : STOKES' THEOREM / DIVERGENCE
THEOREM**

Reading: sections 13.7, 13.8
Homework: see www.courses.fas.harvard.edu/~math21a/

1. STOKES' THEOREM II : TURNING LINE INTEGRALS INTO SURFACE INTEGRALS

(1) Find the line integral of

$$\mathbf{F}(x, y, z) = \langle z, 2x, 3y \rangle$$

around the curve which is the intersection of the plane $z = 2 + x$ with the cylinder $x^2 + y^2 = 1$, oriented counterclockwise as viewed from above.

(2) Find the line integral of

$$\mathbf{F}(x, y, z) = \langle x^2 + y, y^2 + z, z^2 + x \rangle$$

around the triangle with vertices $(2, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 2)$.

2. STOKES' THEOREM III: CUNNING TRICKS

(1) Let

$$\mathbf{F} = x\mathbf{i} + x^3\mathbf{j} + z^2\mathbf{k}$$

(a) Show that

$$\iint_{S_1} \text{curl}(\mathbf{F}) \cdot d\mathbf{S} = \iint_{S_2} \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where S_1 is the part of the paraboloid $z = 4 - x^2 - y^2$ above the xy -plane, oriented upwards, and S_2 is a disc of radius 2 in the xy -plane, oriented upwards.

(Hint: Use Stokes' Theorem twice.)

(b) Compute $\text{curl}(\mathbf{F})$

(c) Compute

$$\iint_{S_2} \text{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

- (2) Use Stokes' Theorem to show that the flux of $\text{curl } \mathbf{F}$ through a closed surface is zero.

3. THE DIVERGENCE THEOREM

- (1) Use the Divergence Theorem to compute the flux of

$$\mathbf{F}(x, y, z) = 3x\mathbf{i} + xy\mathbf{j} + 2xz\mathbf{k}$$

through the surface of the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

- (2) Compute the flux of

$$\mathbf{F}(x, y, z) = ze^{\cos y}\mathbf{i} + y^3\mathbf{j} + 3x^2z\mathbf{k}$$

through the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 2$.

4. A HARDER PROBLEM

- (1) Suppose that $\mathbf{F}(x, y, z)$ is a vector field defined on a simply-connected region of \mathbb{R}^3 which satisfies $\text{curl}(\mathbf{F}) = 0$. Explain why \mathbf{F} is conservative.