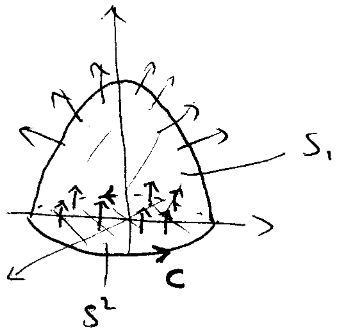
## SECTION ONE

See the solutions to the Stokes' Theorem worksheet from last time.

## SECTION TWO

(1)




(a) The curve C shown in the diagram is the boundary of the surface $S_1$, and is positively oriented, so

$$\iint_{S_1} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$

by Stokes' Theorem. But C is also the boundary of the surface $S_2$, and is positively oriented with respect to the orientation of $S_2$ too, so

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_{S_2} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

by Stokes' Theorem again. Thus $\iint_{S_1} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S} = \iint_{S_2} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$.

(b)

$$\operatorname{curl}(\mathbf{F}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & x^3 & z^2 \end{vmatrix} = \langle 0, 0, 3x^2 \rangle$$

(c) A unit normal vector to $S_2$ is $\mathbf{k}$, and this points upwards (as required) so

$$\iint_{S_2} (\operatorname{curl} \mathbf{F}) \cdot d\mathbf{S} = \iint_{S_2} \langle 0,0,3x^2 \rangle \cdot \langle 0,0,1 \rangle \, dS$$

$$= \iint_{S_2} 3x^2 \, dS$$

$$= \int_{r=0}^{r=2} \int_{\theta=0}^{\theta=2\pi} 3r^2 \cos^2\theta \, r \, d\theta \, dr$$

$$= \underline{\underline{12\pi}}$$