WEDNESDAY 3RD DECEMBER : GREEN'S THEOREM

Reading: sections 13.4 and 13.5
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Evaluating line integrals

(1) Compute
$$\int_C xy\,dx + 2x^2\,dy$$
where C is the line segment from $(-2,0)$ to $(2,0)$ together with the top half of the circle $x^2+y^2=4$, oriented anticlockwise.

(2) Compute
$$\int_C (x^3-y^3)\,dx + (x^3+y^3)\,dy$$
where C is the boundary of the annulus $1 \leq x^2+y^2 \leq 9$.

2. Computing areas

(1) Let D be the region bounded by a simple closed positively-oriented curve C. Use Green's Theorem to compute
   (a) $\int_C x\,dy$
   (b) $-\int_C y\,dx$
   (c) $\frac{1}{2}\int_C x\,dy - y\,dx$
   in terms of the region D.

(2) Compute the area of the region bounded by the hypocycloid parametrized by

$$\mathbf{r}(t) = \cos^3 t\mathbf{i} + \sin^3 t\mathbf{j} \qquad 0 \leq t \leq 2\pi$$

3. A harder problem

Suppose that D is a connected simply-connected region and that the vector field $\mathbf{F} = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ satisfies

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

everywhere. Show that $\mathbf{F}$ is conservative. (Hint: show that the line integral of $\mathbf{F}$ around every closed loop is zero.)