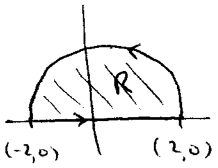
SOLUTIONS : GREEN's THEOREM

SECTION ONE

(1)

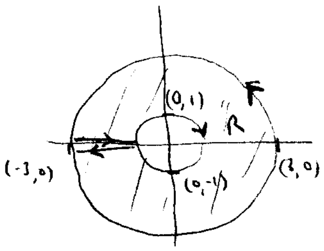


C is positively oriented, so Green's Theorem says

$$\int_C xy\,dx + 2x^2\,dy = \iint_R 4x - x \; dA$$

$$= \iint_R 3x \; dA = \underline{\underline{0}} \quad \text{(symmetry!)}$$

(2)



orienting C as the boundary of the annulus (as shown) we have

$$\int_C (x^3-y^3)\,dx + (x^3+y^3)\,dy$$

$$= \iint_R (3x^2+3y^2)\,dA$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=1}^{r=3} 3r^2 \cdot r\,dr\,d\theta$$

$$= \underline{\underline{120\pi}}$$

SECTION TWO

(1)

$$\int_C x\,dy = -\int_C y\,dx = \frac{1}{2}\int_C x\,dy - y\,dx$$

$$= \iint_D 1 \; dA = \text{area}(D)$$

by Green's Theorem.

(2) $$\text{Area} = \int_C x\,dy = \int_{t=0}^{t=2\pi} \cos^3 t \,.\, 3\sin^2 t \cos t \; dt$$

$$= \int_{t=0}^{t=2\pi} 3\cos^4 t - 3\cos^6 t \; dt = \dots = \underline{\underline{\frac{3\pi}{8}}}$$

SECTION THREE

See p950 of the textbook.