FRIDAY 5TH DECEMBER : CURL AND DIVERGENCE

Reading: sections 13.5, 13.6
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. CURL

(1) Compute

$$\text{curl}(\nabla f)$$

(2) For each of the following vector fields
   (a) compute curl(**F**)
   (b) find f such that $\mathbf{F} = \nabla f$ or explain why no such function f exists
      (i)
$$\mathbf{F}(x, y, z) = e^x \mathbf{i} + e^{2y} \mathbf{j} + e^{3z} \mathbf{k}$$

      (ii)
$$\mathbf{F}(x, y, z) = y^2 \mathbf{i} + x^3 \mathbf{j} + z\mathbf{k}$$

2. Divergence

(1) For each of the following vector fields determine whether
   (a) $\mathbf{F}$ is incompressible
   (b) there is a vector field $\mathbf{G}$ with $\mathbf{F} = \text{curl}(\mathbf{G})$
      (i)
$$\mathbf{F}(x, y, z) = e^x \mathbf{i} + e^{2y} \mathbf{j} + e^{3z} \mathbf{k}$$

      (ii)
$$\mathbf{F}(x, y, z) = y^2 \mathbf{i} + x^3 \mathbf{j} + z \mathbf{k}$$