SOLUTIONS: CURL AND DIVERGENCE

SECTION ONE

(1) $\quad \operatorname{curl}(\nabla f) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix}$

$$= \left\langle \frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}, \frac{\partial^2 f}{\partial z \partial x} - \frac{\partial^2 f}{\partial x \partial z}, \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \right\rangle$$

$$= \langle 0, 0, 0 \rangle$$ by Clairault's Theorem.

(2) (i) (a) $\quad \operatorname{curl} \underset{\sim}{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x & e^{2y} & e^{3z} \end{vmatrix} = \langle 0, 0, 0 \rangle$

(b) $\quad \underset{\sim}{F} = \nabla f$ where $f(x,y,z) = e^x + \frac{1}{2}e^{2y} + \frac{1}{3}e^{3z}$

(ii) (a) $\quad \operatorname{curl} \underset{\sim}{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & x^3 & z \end{vmatrix} = \langle 0, 0, 3x^2 - 2y \rangle$

(b) There is no function f with $\underset{\sim}{F} = \nabla f$, as $\operatorname{curl}(\nabla f) = 0$ but $\operatorname{curl} \underset{\sim}{F} \neq 0$.

SECTION TWO

(1)(i)(a) $\quad \operatorname{div} \underset{\sim}{F} = e^x + 2e^{2y} + 3e^{3z}$

$\neq 0$ so $\underset{\sim}{F}$ is <u>not</u> incompressible

(b) Since $\operatorname{div}(\operatorname{curl} \underset{\sim}{G}) = 0$ for any vector field $\underset{\sim}{G}$, $\underset{\sim}{F}$ is not the curl of any vector field.

(ii) (a) $\quad \operatorname{div} \underset{\sim}{F} = \frac{\partial}{\partial x}(y^2) + \frac{\partial}{\partial y}(x^3) + \frac{\partial}{\partial z}(z) = 1$

$\neq 0$ so $\underset{\sim}{F}$ is <u>not</u> incompressible.

(b) Since $\operatorname{div} \underset{\sim}{F} \neq 0$, $\underset{\sim}{F}$ is not equal to $\operatorname{curl} \underset{\sim}{G}$ for any $\underset{\sim}{G}$.