SECTION TWO

(1) Parametrize S by $x = u$ $y = v$ $z = \sqrt{u^2+v^2}$ where $(u,v) \in R = \{0 \le u^2+v^2 \le 1\}$. Then $\underset{\sim}{r}_u = \langle 1, 0, \frac{u}{\sqrt{u^2+v^2}} \rangle$

$\underset{\sim}{r}_v = \langle 0, 1, \frac{v}{\sqrt{u^2+v^2}} \rangle$

and so $\underset{\sim}{r}_u \times \underset{\sim}{r}_v = \langle -\frac{u}{\sqrt{u^2+v^2}}, -\frac{v}{\sqrt{u^2+v^2}}, 1 \rangle$.

This orientation points upward (look at the $\underset{\sim}{k}$-component) so

$$\iint_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = -\iint_R \langle u, v, (u^2+v^2)^2 \rangle \cdot \langle -\frac{u}{\sqrt{u^2+v^2}}, -\frac{v}{\sqrt{u^2+v^2}}, 1 \rangle \, du\, dv$$

$$= \iint_R \left(\sqrt{u^2+v^2} - (u^2+v^2)^2 \right) du\, dv$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} (r - r^4)\, r\, dr\, d\theta$$

$$= 2\pi \left(\tfrac{1}{3} - \tfrac{1}{6} \right) = \underline{\underline{\pi/3}}$$

(2) The ~~normal~~ outward-pointing normal to S at the point (x,y,z) on S is $\frac{1}{2}\langle x, y, z \rangle$. (We use the outward-pointing normal as S is a closed surface)

Thus

$$\iint_S \underset{\sim}{F} \cdot d\underset{\sim}{S} = \iint_S (\underset{\sim}{F} \cdot \underset{\sim}{n})\, dS$$

$$= \iint_S \langle x, y, z \rangle \cdot \langle \tfrac{x}{2}, \tfrac{y}{2}, \tfrac{z}{2} \rangle \, dS$$

$$= \frac{1}{2} \iint_S x^2+y^2+z^2 \, dS$$

$$= \frac{1}{2} \iint_S 4 \, dS = 2 . \text{Area}(S)$$

$$= \underline{\underline{32\pi}}$$