# MONDAY 10TH NOVEMBER : DOUBLE INTEGRALS

## 1. Estimating double integrals

(1) Let $R$ be the rectangle $[0, 3] \times [0, 3]$. Let $f(x, y) = 6 + x - y$. Use a $3 \times 3$ grid of squares with side 1 to estimate

$$\iint_R f(x, y) \, dA$$

## 2. Evaluating double integrals

(1) Evaluate

$$\iint_R \sqrt{4 - x^2} \, dA$$

where $R = [-2, 2] \times [-3, 3]$, by interpreting it as a volume.

(2) Compute the mass of a circular disc of radius 3m with center the origin and density per square meter

$$\rho(x, y) = \sqrt{9 - x^2 - y^2}$$

at the point $(x, y)$.

(3) Mohammed's Mountain Moving Company have contracted to move a small hill from Point A to Point B. The hill has a circular base (of radius 10m and center the origin) and the height (in meters) above the point $(x, y)$ is $100 - \sqrt{x^2 + y^2}$. What volume of rock do they have to move?

## 3. A HARDER PROBLEM

(1) Show that the integral of the function
$$f(x) = \begin{cases} 2x & x \notin \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\} \\ x^2 & x \in \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\} \end{cases}$$
over the interval $[0, 1]$ exists. Compute it.