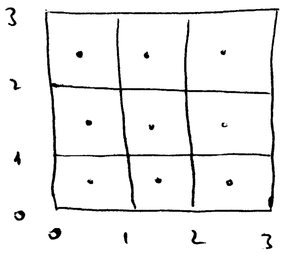
SOLUTIONS: DOUBLE INTEGRALS

SECTION ONE

(1) 

Take sample points in the middle of the squares.

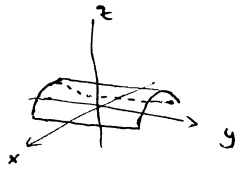
Then $\iint_R f(x,y)\,dA \approx \sum_{i=1}^{3}\sum_{j=1}^{3} f(i-\tfrac{1}{2}, j-\tfrac{1}{2})\,\Delta A$

$= 6+7+8+5+6+7+4+5+6$

$= \underline{\underline{54}}$

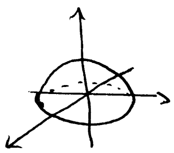
SECTION TWO

(1) The surface $z=\sqrt{4-x^2}$ is the top half of a cylinder of radius 2 centered along the y-axis, so



$\iint_{[-2,2]\times[-3,3]} \sqrt{4-x^2}\,dA$ is the volume of half of a solid cylinder with radius 2 and height 6, i.e. $\frac{1}{2}\pi r^2 h = \underline{\underline{12\pi}}$

(2) The mass of the disc D is $\iint_D \sqrt{9-x^2-y^2}$, which can also be interpreted as the volume above D and below the surface $z=\sqrt{9-x^2-y^2}$. This surface is the top half of a phere of radius 3 about the origin, so



$\iint_D \sqrt{9-x^2-y^2}$ is the volume of half a solid sphere of radius 3, i.e. $2/3\,\pi r^3 = \underline{\underline{18\pi}}$.

(3) The hill is a right circular cone of radius 10m and height 100m, so has volume $\frac{1}{3}\pi r^2 h = \underline{\underline{\frac{10000\pi}{3}}}$

SECTION THREE

Let $g(x)=2x$. Form a Riemann sum I_n for $\int_0^1 f(x)\,dx$ by dividing $[0,1]$ into n equal regions, so $I_n = \sum_{i=1}^{n} f(x_i^*)\cdot\frac{1}{n}$ where x_i^* is the sample point between $\frac{i-1}{n}$ and $\frac{i}{n}$. Let $J_n = \sum_{i=1}^{n} g(x_i^*)\frac{1}{n}$ be the corresponding Riemann sum for $\int_0^1 g(x)\,dx$. Then $|I_n - J_n| \le \frac{2}{n}\left(\frac{1}{2}-\frac{1}{4^2}+1-\frac{1}{2^2}+\frac{3}{2}-(\frac{3}{4})^2\right) = 17/4n$ because f and g only differ at $x=\frac{1}{4}$, $x=\frac{1}{2}$ and $x=3/4$ and each of these points is in at most two subintervals. Thus $|I_n - J_n|\to 0$ as $n\to\infty$, so $\lim_{n\to\infty} I_n = \lim_{n\to\infty} J_n = 1$, and so $f(x)$ is integrable over $[0,1]$.