FRIDAY 14TH NOVEMBER : DOUBLE INTEGRALS III

Reading: sections 12.3 and 12.4
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Evaluating double integrals

(1) Let R be the region bounded by $y = x - x^2$ and $y = \frac{x}{4}$. Compute

$$\iint_R xy \, dA$$

(2) Let R be the lower region bounded by $y = \sqrt{x}$, $y = x^4$ and $y = \frac{1}{2}$. Compute

$$\iint_R y^2 \, dA$$

(3) Compute the volume bounded by the parabolic cylinders $y = x^2$, $y = (x-2)^2$ and $z = 9 - x^2$, the xz-plane and the xy-plane.

2. Changing the order of integration

(1) Express

$$\int_{x=0}^{x=1} \int_{y=x^2}^{y=\sqrt[4]{x}} f(x,y)\, dy\, dx$$

as an iterated integral in the other order.

(2) Compute

$$\int_{y=0}^{y=4} \int_{x=\sqrt{y}}^{x=2} e^{x^3}\, dx\, dy$$

3. Polar co-ordinates

(1) Compute the volume of the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{4} + z^2 = 1$$

(2) Compute the volume below the sphere $x^2 + y^2 + z^2 = 1$ and above the sphere $x^2 + y^2 + (z-1)^2 = 1$.