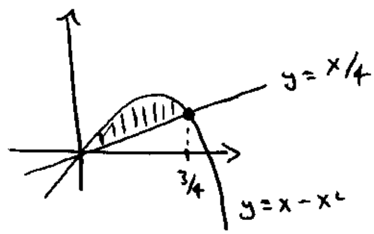
SOLUTIONS : DOUBLE INTEGRALS III

SECTION ONE

(1)

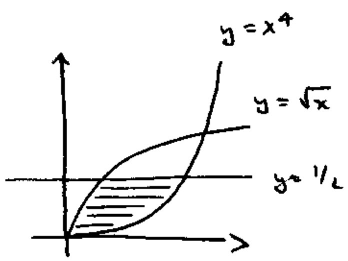


$$\iint_R xy\,dA = \int_{x=0}^{x=3/4}\int_{y=x/4}^{y=x-x^2} xy\,dy\,dx$$

$$= \int_{x=0}^{x=3/4} \frac{x^3(15-32x+16x^2)}{32}\,dx$$

$$= \underline{\underline{\frac{729}{163840}}}$$

(2)

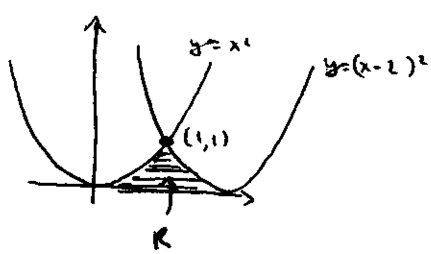


$$\iint_R y^2\,dA = \int_{y=0}^{y=\frac{1}{2}}\int_{x=y^2}^{x=\sqrt[4]{y}} y^2\,dx\,dy$$

$$= \int_{y=0}^{y=1/2} y^{9/4} - y^4\,dy$$

$$= \underline{\underline{\frac{1}{26.\,2^{1/4}} - \frac{1}{180}}}$$

(3)



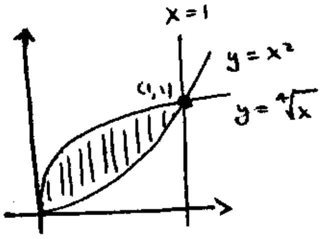
$$\text{volume} = \iint_R 9 - x^2\,dA$$

$$= \int_{y=0}^{y=1}\int_{x=\sqrt{y}}^{x=2-\sqrt{y}} 9-x^2\,dx\,dy$$

$$= \underline{\underline{\frac{79}{15}}}$$

SECTION TWO

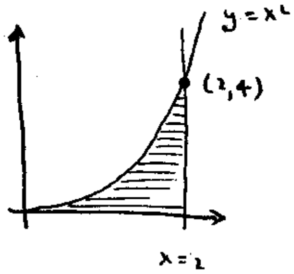
(1)



$$\int_{x=0}^{x=1}\int_{y=x^2}^{y=\sqrt[4]{x}} f(x,y)\,dy\,dx$$

$$= \int_{y=0}^{y=1}\int_{x=y^4}^{x=\sqrt{y}} f(x,y)\,dx\,dy$$

(2)



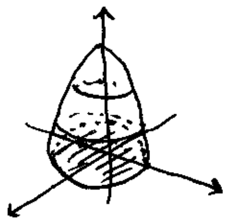
$$\int_{y=0}^{y=4}\int_{x=\sqrt{y}}^{x=2} e^{x^3}\,dx\,dy$$

$$= \int_{x=0}^{x=2}\int_{y=0}^{y=x^2} e^{x^3}\,dy\,dx$$

$$= \int_{x=0}^{x=2} x^2 e^{x^3}\,dx = \underline{\underline{\tfrac{1}{3}\left(e^8-1\right)}}$$

SECTION THREE

(1)



HALF OF THE ELLIPSOID

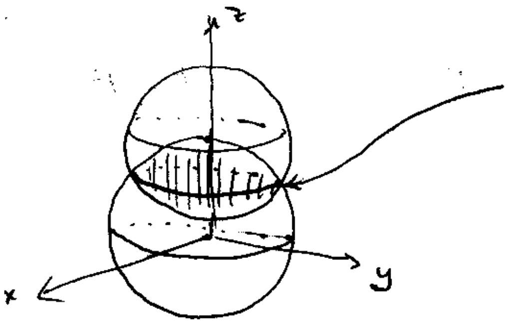
Volume of ellipsoid $= 2\times$ volume of half the ellipsoid

$$= 2\iint_D \sqrt{1-\frac{x^2}{4}-\frac{y^2}{4}}\;dA$$

where $D =$ disc of radius 2 about the origin

$$\text{Volume} = 2\int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=2}\sqrt{1-r^2/4}\;r\,dr\,d\theta = \underline{\underline{\frac{16\pi}{3}}}$$

(2)



the spheres meet when $x^2+y^2+z^2=1$ and $x^2+y^2+z^2-2z+1=1$

$\Rightarrow 1-2z=0 \quad\Rightarrow\quad z=\frac{1}{2}$

and so $x^2+y^2 = 3/4$ (a circle of radius $\frac{\sqrt{3}}{2}$)

$$\text{Volume} = \iint_{\text{base}} \text{height}\;dA = \iint_D \sqrt{1-x^2-y^2} - \left(1-\sqrt{1-x^2-y^2}\right)dA$$

where D is a disc centred at the origin of radius $\frac{\sqrt{2}}{2}$

$$\text{so Volume} = \iint_D 2\sqrt{1-x^2-y^2} - 1\;dA = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=\sqrt{3}/2}\left(2r\sqrt{1-r^2}-r\right)dr\,d\theta = \frac{5\pi}{12}$$