MONDAY 17TH NOVEMBER : POLAR CO-ORDINATES / SURFACE AREA

Reading: sections 12.6 and 12.7
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Polar Co-ordinates

(1) Find the volume of the solid bounded by the paraboloid $z = 10 - x^2 - y^2$ and the plane $z = 6$.

(2) Find the volume under the plane $z = 1 + x + y$ and above the disc of radius 1 centered at $(0, 1)$.

2. Surface area

(1) Find an equation for the tangent plane to the helicoid parametrized by

$$\mathbf{r}(u, v) = u \cos v\mathbf{i} + u \sin v\mathbf{j} + v\mathbf{k}$$

at the point $(x, y, z) = (4, 0, 2\pi)$. Find the surface area of the part of the helicoid where $0 \leq u \leq 1$ and $0 \leq v \leq 2\pi$.

(2) Find the area of the part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = 1$.

(3) Find the area of the part of the surface $x = \frac{1}{2}y + \frac{1}{2}z^2$ inside the cube $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$.