WEDNESDAY 19TH NOVEMBER : SURFACE AREA / TRIPLE INTEGRALS

Reading: sections 12.6 and 12.7
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Surface Area

(1) Find the area of the part of the surface $x = y^2 - z^2$ inside the cylinder $y^2 + z^2 = 1$.

(2) Show that the area of the part of the graph $z = f(x, y)$ above the region D in the xy-plane is

$$\iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$

2. Triple Integrals

(1) Let E be the region bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$. Compute

$$\iiint_E x \, dV$$

(2) Find the mass of the region bounded by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ if the density at the point (x, y, z) is x^2.

3. A harder problem

(1) Suppose that setting

$$x = g(u, v) \qquad y = h(u, v)$$

(where g and h are smooth functions) produces a one-to-one correspondence between the region S in the uv-plane and the region R in the xy-plane. Write

$$\iint_R f(x, y) \, dA$$

as an integral over S.