FRIDAY 21ST NOVEMBER : TRIPLE INTEGRALS

Reading: sections 12.7 and 12.8
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Triple Integrals

(1) Sketch the region of integration for

$$\int_{x=0}^{x=1} \int_{y=\sqrt{x}}^{y=1} \int_{z=0}^{z=1-y} f(x,y,z)\, dz\, dy\, dx$$

(2) Compute

$$\iiint_E z\, dV$$

where E is the region in the first octant bounded by the planes $y+z=1$ and $x+z=1$.

(3) Express

$$\iiint_E x\, dV$$

as an iterated integral, where E is the region in the first octant bounded by the surfaces $y=3z$ and $x^2+y^2=9$.

2. Polar co-ordinates

(1) Evaluate

$$\iiint_E y\,dV$$

where E is the region between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ which lies above the xy-plane and below the plane $z = x + 2$.

(2) Find the mass of the region

$$\{(x, y, z) : 1 \leq x^2 + y^2 + z^2 \leq 4, z \geq 0\}$$

if the density at the point (x, y, z) is z.

3. A harder problem

(1) Suppose that setting

$$x_1 = f(u_1, u_2, u_3) \qquad x_2 = g(u_1, u_2, u_3) \qquad x_3 = h(u_1, u_2, u_3)$$

(where f, g and h are smooth functions) produces a one-to-one correspondence between the region D in (u_1, u_2, u_3)-space and the region E in (x_1, x_2, x_3)-space. Let

$$G(u_1, u_2, u_3) = F(f(u_1, u_2, u_3), g(u_1, u_2, u_3), h(u_1, u_2, u_3))$$

Show that

$$\iiint_E F(x_1, x_2, x_3)\,dV = \iiint_D G(u_1, u_2, u_3)\,|\det J|\,dV$$

where the *Jacobian matrix* J has (i, j) entry equal to $\frac{\partial x_i}{\partial u_j}$.