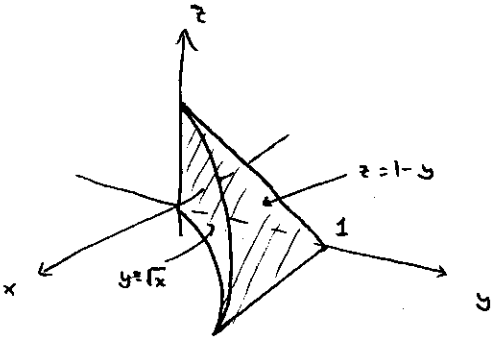
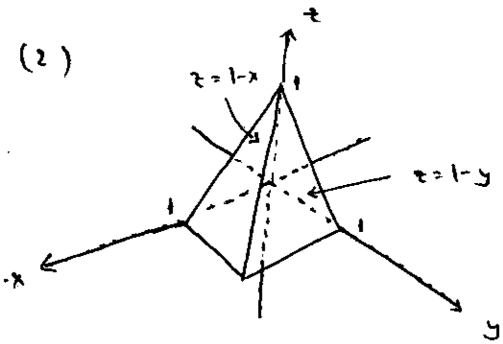
SOLUTIONS : TRIPLE INTEGRALS

SECTION ONE

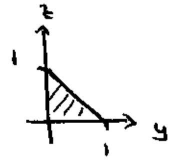
(1)



(2)



Projecting to the yz-plane, we see



so

$$\iiint_E z \, dV = \int_{y=0}^{y=1} \int_{z=0}^{z=1-y} \int_{x=0}^{x=1-z} z \, dx \, dz \, dy$$

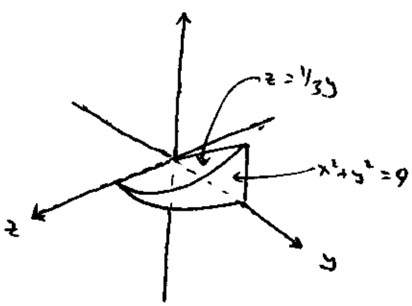
$$= \int_{y=0}^{y=1} \int_{z=0}^{z=1-y} z - z^2 \, dz \, dy$$

$$= \int_{y=0}^{y=1} \tfrac{1}{2}(1-y)^2 - \tfrac{1}{3}(1-y)^3 \, dy$$

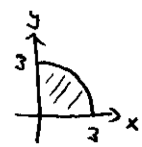
$$= \left[\tfrac{1}{12}(1-y)^4 - \tfrac{1}{6}(1-y)^3 \right]_{y=0}^{y=1}$$

$$= 1/6 - 1/12 = \underline{\underline{1/6}}$$

(3)



Projecting this region to the xy-plane we see

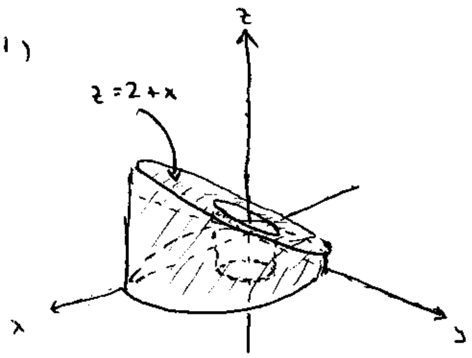


so

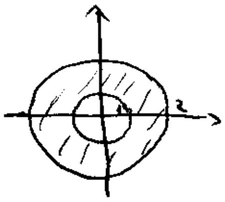
$$\iiint_E x \, dV = \int_{x=0}^{x=3} \int_{y=0}^{y=\sqrt{9-x^2}} \int_{z=0}^{z=1/3 y} x \, dz \, dy \, dx$$

SECTION TWO

(1)



Projecting this region to the xy-plane, we see



so, using cylindrical polar co-ordinates on $\mathbb{R}^3$, we have:

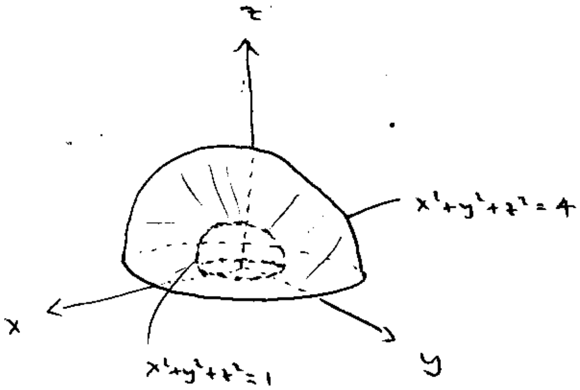
$$\iiint_E y\,dV = \int_{\theta=0}^{\theta=2\pi}\int_{r=1}^{r=2}\int_{z=0}^{z=2+r\cos\theta} r\sin\theta\; r\,dz\,dr\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{r=1}^{r=2} 2r^2\sin\theta + \tfrac{1}{2}r^3\sin 2\theta \;dr\,d\theta$$

$$= \cdots = \underline{\underline{0}}$$

(which is also obvious from symmetry considerations)

(2)



$$\text{Mass} = \iiint_E z\,dV$$

$$= \int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/2}\int_{\rho=1}^{\rho=2} \rho^3\sin\phi\cos\phi\;d\rho\,d\phi\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\int_{\phi=0}^{\phi=\pi/2} \frac{15}{8}\sin 2\phi\;d\phi\,d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi}\left[\frac{15}{16}\cos 2\phi\right]_{\phi=\pi/2}^{\phi=0} d\theta$$

$$= \underline{\underline{\frac{15\pi}{4}}}$$

SECTION THREE

See section 12.9 of the textbook.