MONDAY 24TH NOVEMBER : TRIPLE INTEGRALS / VECTOR FIELDS

Reading: sections 13.1 and 13.2
Homework: see `www.courses.fas.harvard.edu/~math21a/`

1. Triple Integrals

(1) Compute

$$\iiint_E y^2 z^2 \, dV$$

where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

(2) Evaluate

$$\iiint_E z^3 \sqrt{x^2 + y^2 + z^2} \, dV$$

where E is the upper solid hemisphere bounded by the xy-plane and the sphere of radius 1 about the origin.

2. Vector Fields

(1) Sketch the vector field given by

$$\mathbf{F}(x,y) = \left(-\frac{y}{\sqrt{x^2+y^2}}\right)\mathbf{i} + \left(\frac{x}{\sqrt{x^2+y^2}}\right)\mathbf{j}$$

Sketch some flowlines for this vector field.

(2) Sketch the vector field ∇f, where

$$f(x,y) = 1 - x^2 - y^2$$

Sketch some flowlines for this vector field.

(3) Explain why flowlines of the *conservative vector field* ∇f never pass through critical points of the function $f(x,y)$. Can flowlines of ∇f be closed loops?