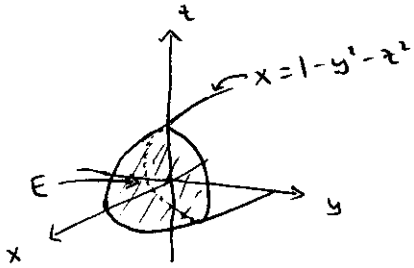
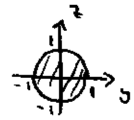
SOLUTIONS: TRIPLE INTEGRALS / VECTOR FIELDS

SECTION ONE

(1)



Projecting E to the yz-plane, we see



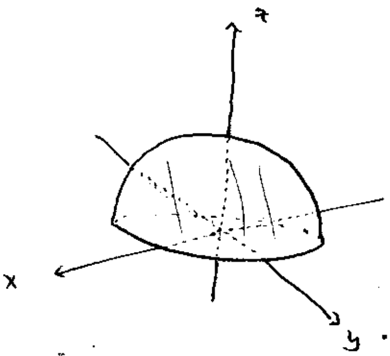
and so

$$\iiint_E y^2 z^2 \, dV = \iint_{disc} \left(\int_{x=0}^{x=1-y^2-z^2} y^2 z^2 \, dx \right) dy\, dz$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=1} r^4 \sin^2\theta \cos^2\theta \, (1-r^2)\, r\, dr\, d\theta$$

$$= \frac{1}{96} \int_{\theta=0}^{\theta=2\pi} \sin^2 2\theta \, d\theta = \underline{\underline{\pi/96}}$$

(2)



$$\iiint_E z^3 \sqrt{x^2+y^2+z^2} \, dV = \int_{\theta=0}^{\theta=2\pi} \int_{\phi=0}^{\phi=\pi/2} \int_{\rho=0}^{\rho=1} \rho^6 \cos^3\phi \sin\phi \, d\rho\, d\phi\, d\theta$$

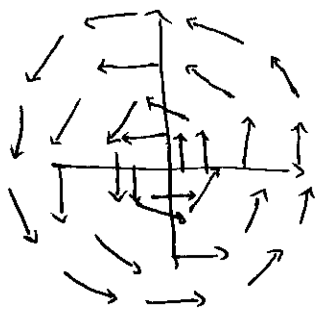
$$= \int_{\theta=0}^{\theta=2\pi} \int_{\phi=0}^{\phi=\pi/2} \frac{1}{7} \cos^3\phi \sin\phi \, d\phi\, d\theta$$

$$= \int_{\theta=0}^{\theta=2\pi} \left[\frac{1}{28} \cos^4\phi \right]_{\phi=\pi/2}^{\phi=0} d\theta$$

$$= \underline{\underline{\pi/14}}$$

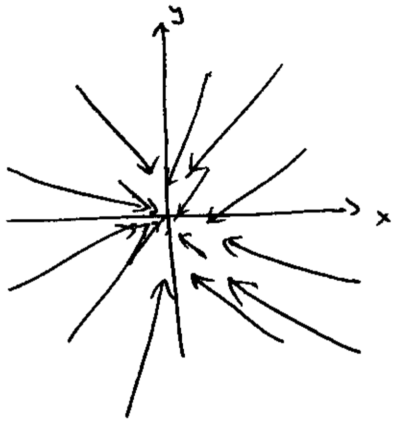
SECTION TWO

(1)



Flowlines are circles with center the origin.

(2) $\nabla f = \langle -2x, -2y \rangle$



Flow lines are radial straight lines.

(3) Flowlines of ∇f never pass through critical points of f for the following reason. Flowlines represent the path of a particle moving with velocity equal to the vector field. Critical points of f are where $\nabla f = \underset{\sim}{0}$, so ∇f is a very small vector near a critical point of f. Thus the particle slows down as it approaches a critical point; it never actually reaches the critical point and so flowlines don't go through critical points.

Since ∇f points in the direction of steepest increase of f, the value of f increases as we follow a flowline of ∇f. Thus flowlines of ∇f can never be closed loops.