

WEDNESDAY 26TH NOVEMBER : LINE INTEGRALS

Reading: sections 13.2 and 13.3

Homework: see www.courses.fas.harvard.edu/~math21a/

1. INTEGRALS OF FUNCTIONS ALONG CURVES

- (1) Compute the mass of the wire which runs along the part of the parabola $x = y^2$ from $(0, 0)$ to $(4, 2)$ if the density at the point (x, y) is y .

- (2) Compute the area below the surface $z = 36x^3$ and above the part of the curve $y = x^3$ in the xy -plane from $(0, 0)$ to $(1, 1)$.

2. INTEGRALS OF VECTOR FIELDS ALONG CURVES

(1) Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the curve parametrized by $t^3\mathbf{i} + t^2\mathbf{j}$, $0 \leq t \leq 2$ and $\mathbf{F}(x, y) = xy^2\mathbf{i} - \sqrt{y}\mathbf{j}$.

(2) Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the part of the parabola $y = 1 - x^2$ from $(-1, 0)$ to $(1, 0)$ and $\mathbf{F}(x, y) = x^2\mathbf{i} + y^2\mathbf{j}$.

(3) Let C be a curve parametrized by $(x(t), y(t))$, $a \leq t \leq b$, and let $\mathbf{F}(x, y) = \nabla f(x, y)$.
What is

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

in terms of the function f ?